

**94-775/95-865 Lecture 3: Finding  
Possibly Related Entities,  
Visualizing High-Dimensional  
Vectors**

George Chen

# Last Time: Co-Occurrences

- Joint probability  $P(A, B)$  can be poor indicator of whether A and B co-occurring is “interesting”
- Find interesting relationships between pairs of items by looking at PMI
  - Intuition: “Interesting” co-occurring events should occur more frequently than if they were to co-occur independently
- Find interesting relationship between *types* of items (and *not* specific pairs of items) using chi-square (or equivalently phi-square)

# Co-occurrence Analysis Applications

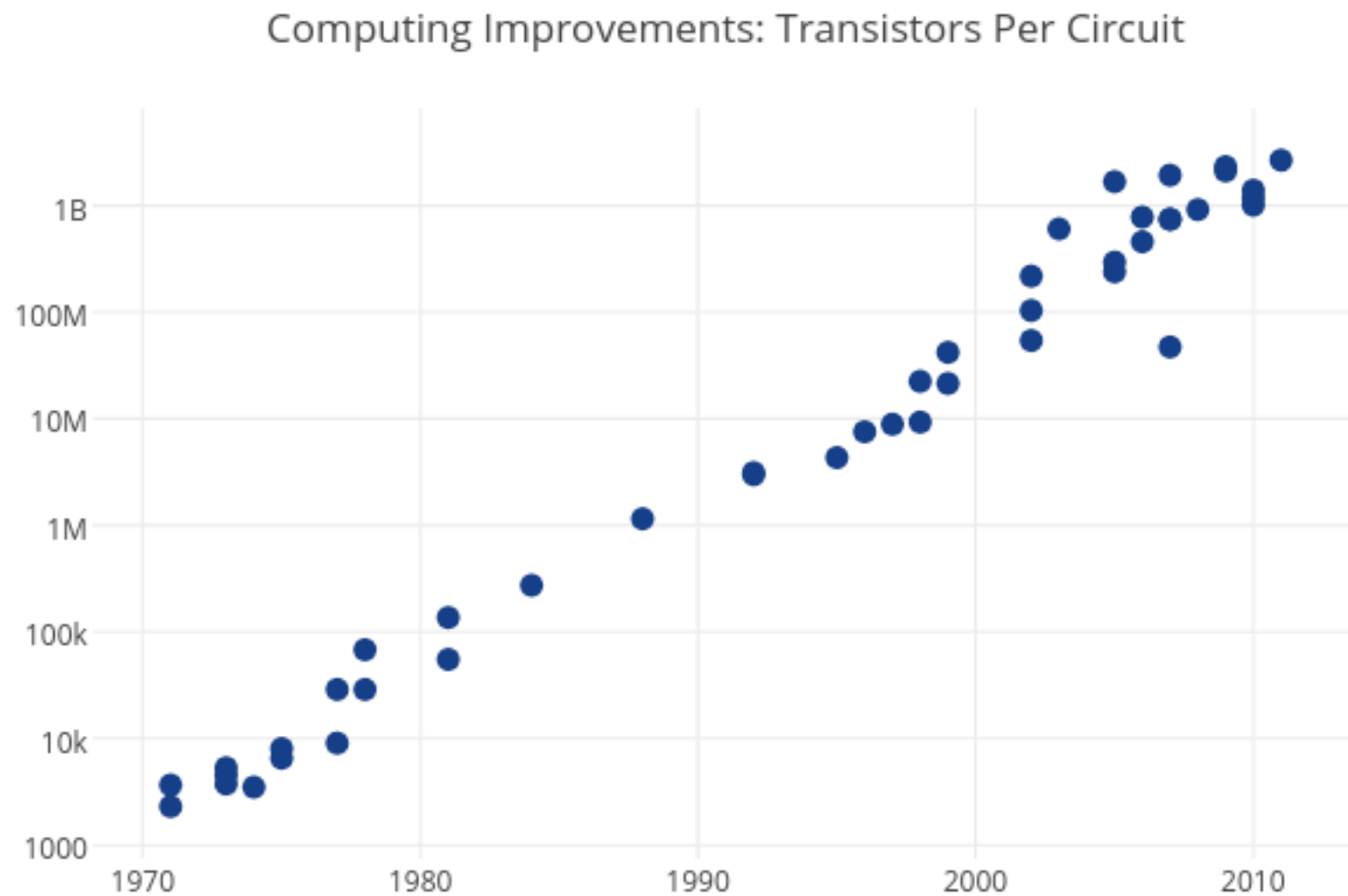
- If you're an online store/retailer:  
anticipate *when* certain products are likely to be purchased/  
rented/consumed more
  - Products & dates
- If you have a bunch of physical stores:  
anticipate *where* certain products are likely to be purchased/  
rented/consumed more
  - Products & locations
- If you're the police department:  
create "heat map" of where different criminal activity occurs
  - Crime reports & locations

# Co-occurrence Analysis Applications

- If you're an online store/retailer:
  - anticipate when certain products are likely to be purchased/returned
- Examples of data to take advantage of:
  - data collected by your organization
  - social networks
  - news websites
  - blogs
- If you are an online store/retailer:
  - Web scraping frameworks can be helpful:
    - Scrapy
    - Selenium (great with JavaScript-heavy pages)
- If you are a crime analyst:
  - Crime reports & locations

# Continuous Measurements

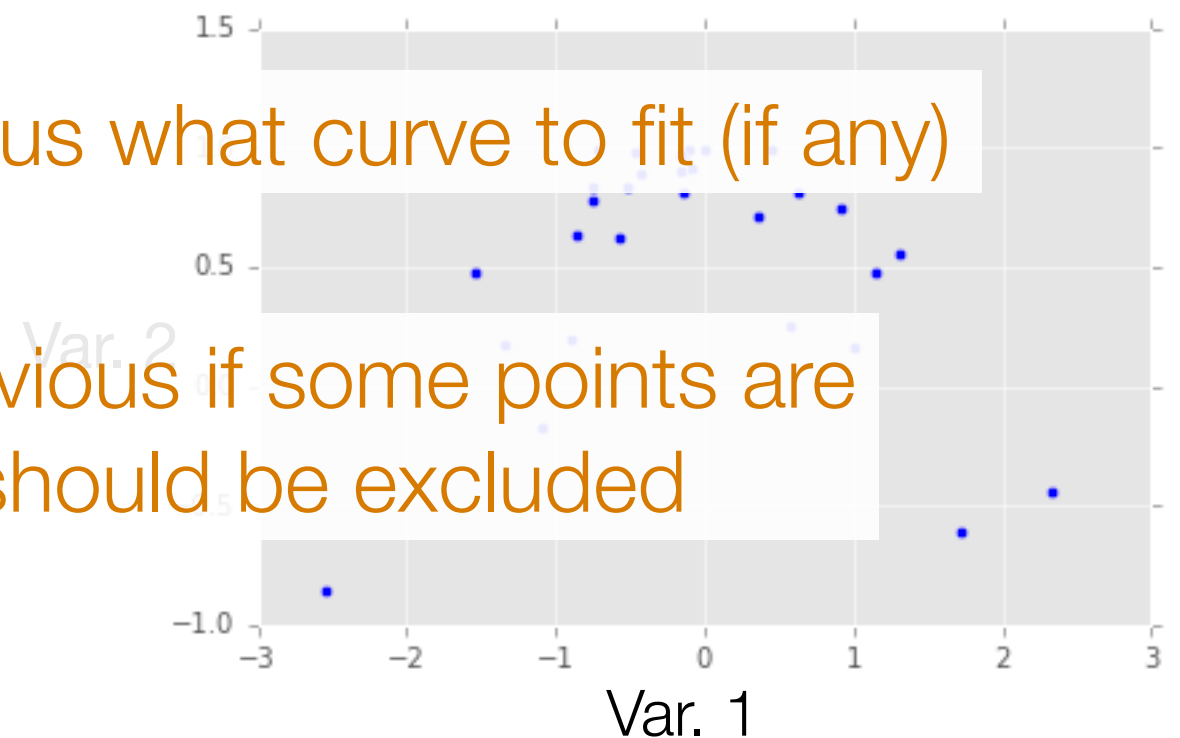
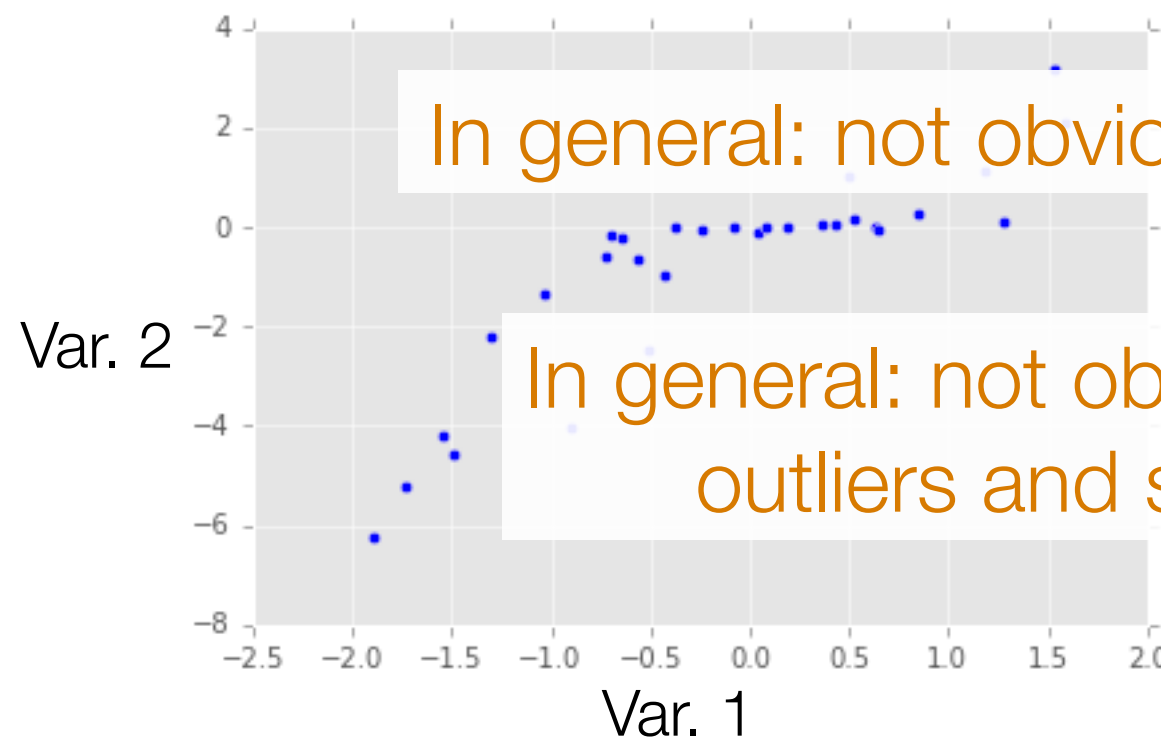
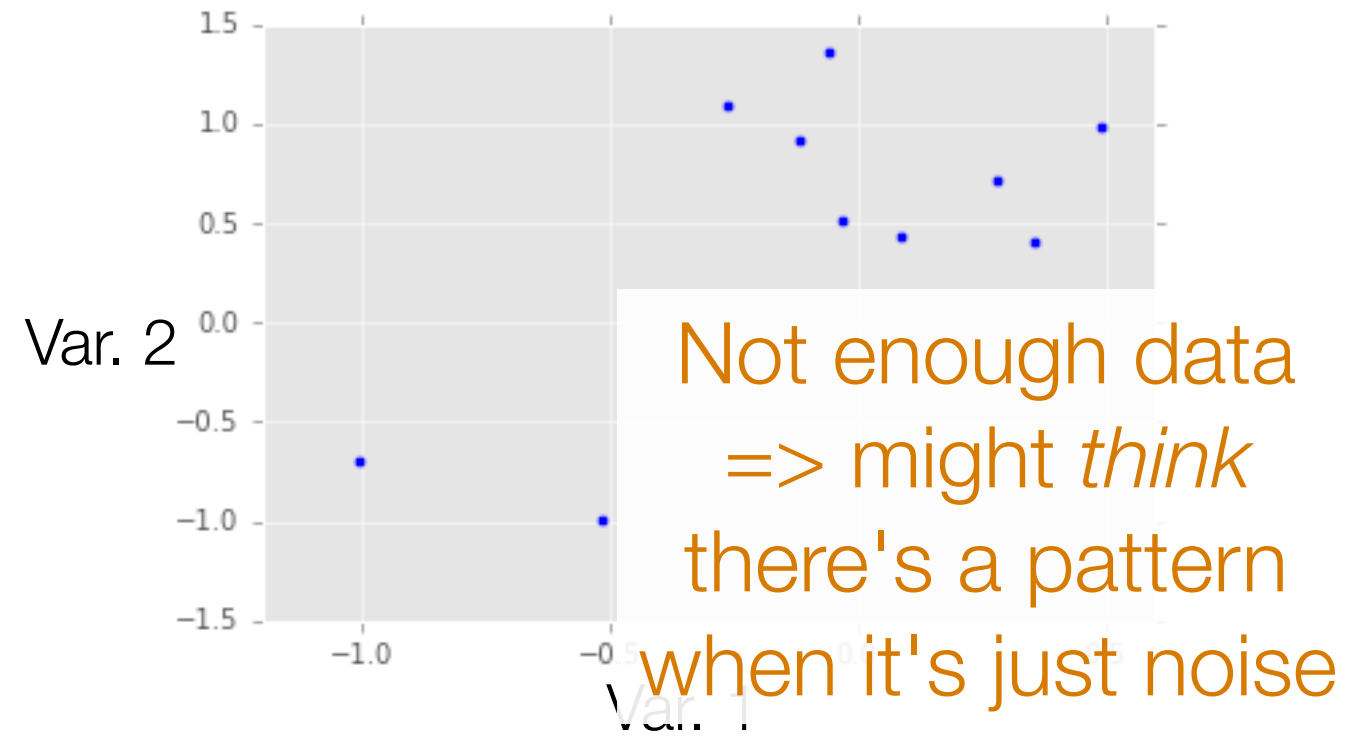
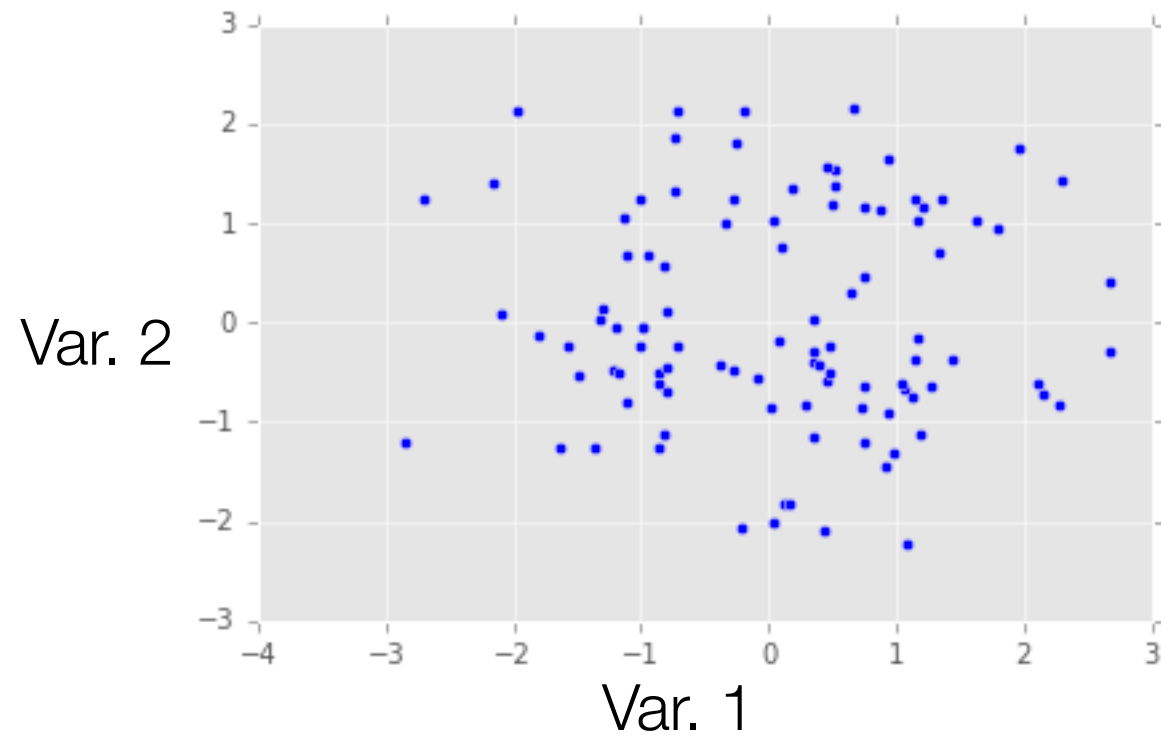
- So far, looked at relationships between *discrete* outcomes
- For pair of *continuous* outcomes, use a **scatter plot**



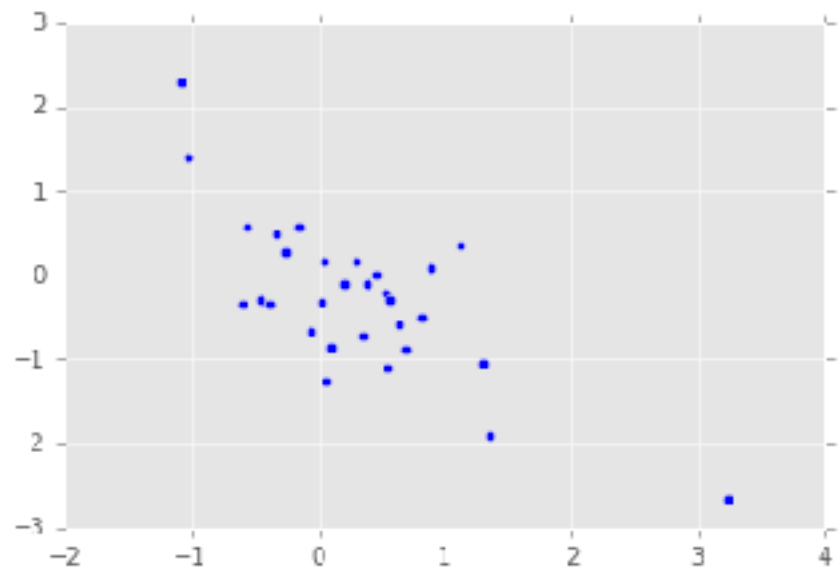
Of course, not all trends look like a line

(so don't just do linear regression!)

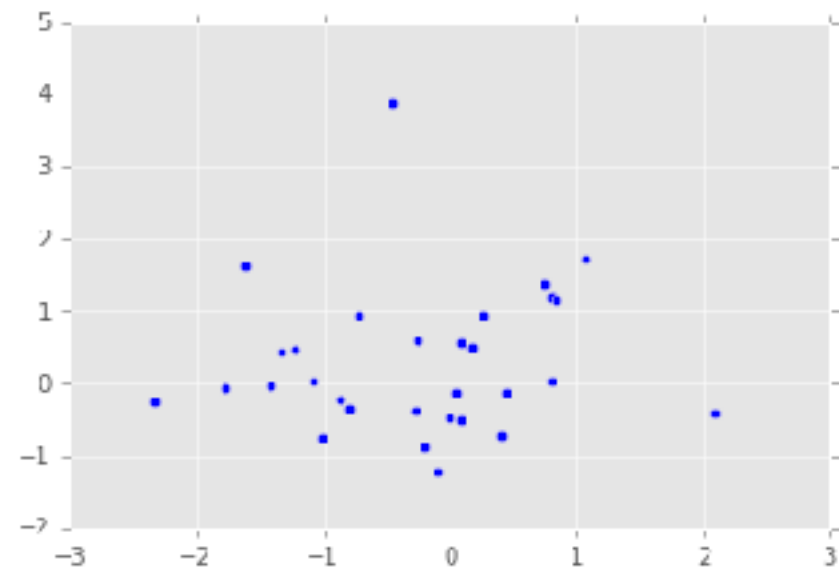
# The Importance of Staring at Data



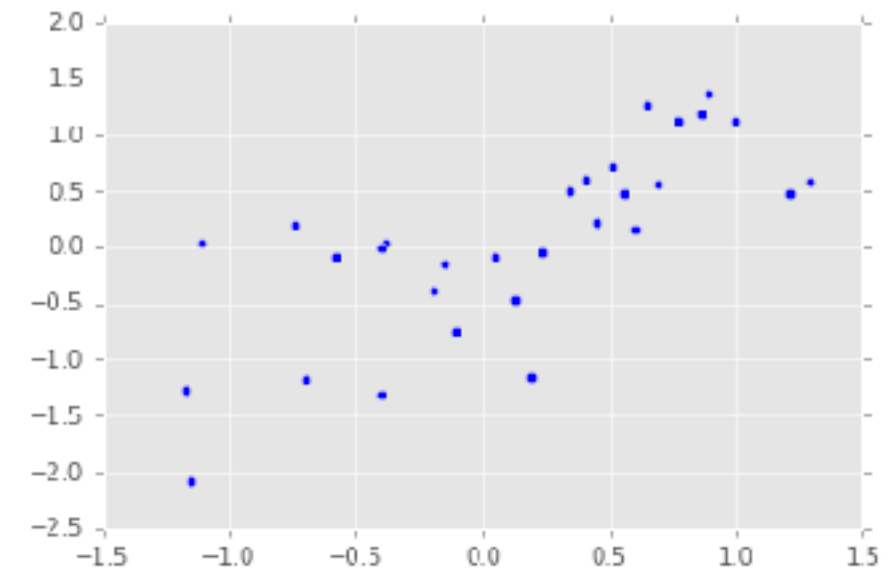
# Correlation



Negatively correlated



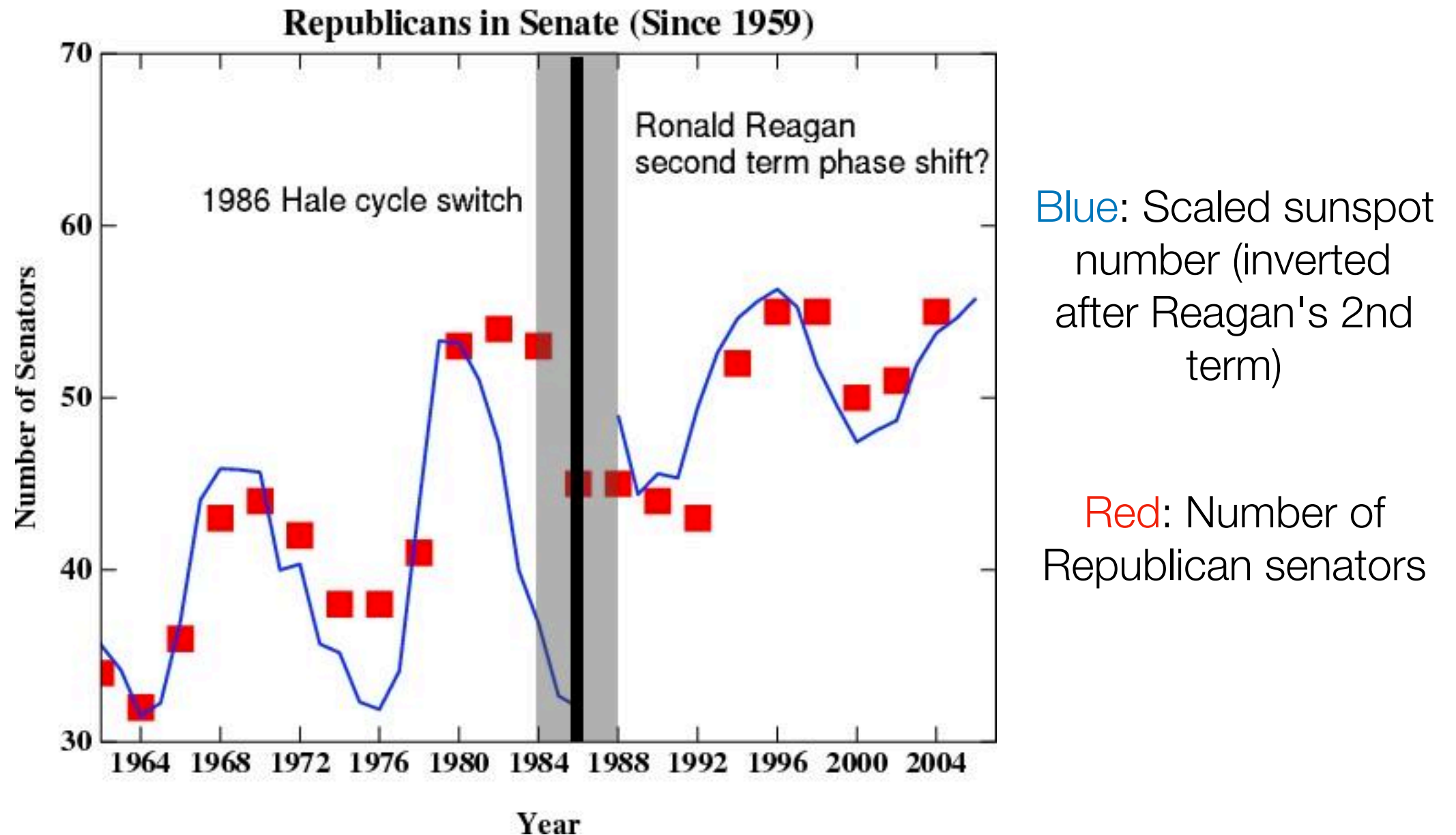
Not really correlated



Positively correlated

Beware: Just because two variables appear correlated doesn't mean that one can predict the other

# Correlation $\neq$ Causation



Moreover, just because we find correlation in data doesn't mean it has predictive value!



**Important: At this point in the course, we are finding *possible* relationships between two entities**

We are *not* yet making statements about prediction (we'll see prediction later in the course)

We are *not* making statements about causality (beyond the scope of this course)

# Causality



Studies in 1960's: Coffee drinkers have higher rates of lung cancer

*Can we claim that coffee is a cause of lung cancer?*

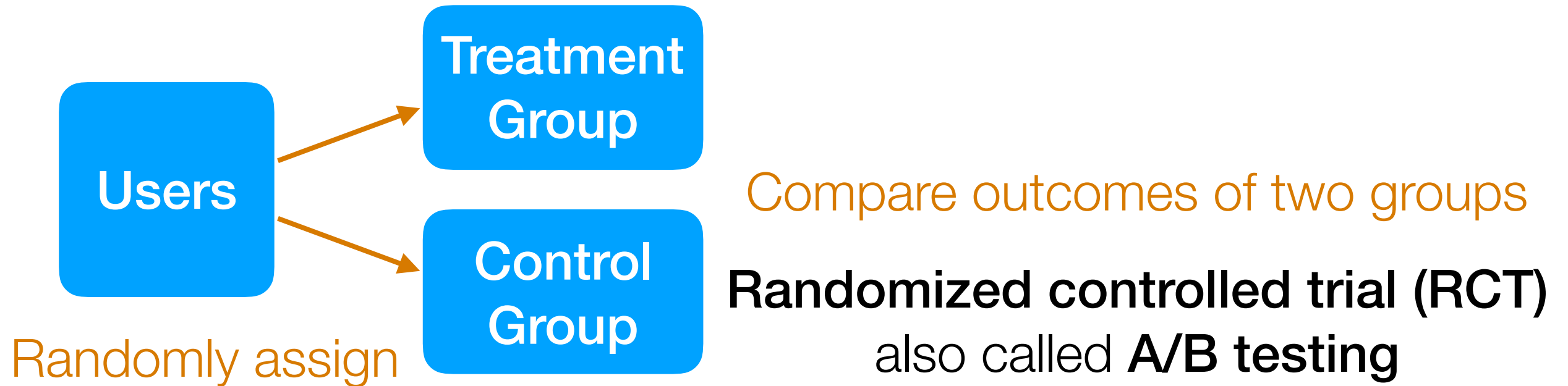
Back then: coffee drinkers also tended to smoke more than non-coffee drinkers (smoking is a **confounding variable**)

To establish causality, groups getting different treatments need to appear similar so that the only difference is the treatment

Image source: George Chen

# Establishing Causality

If you control data collection



Example: figure out webpage layout to maximize revenue (Amazon)

Example: figure out how to present educational material to improve learning (Khan Academy)

If you do not control data collection

In general: *not* obvious establishing what caused what

# 94-775/95-865

## Part I: Exploratory data analysis

*Identify structure present in “unstructured” data*

- Frequency and co-occurrence analysis *Basic probability & statistics*
- Visualizing high-dimensional data/dimensionality reduction
- Clustering
- Topic modeling (a special kind of clustering)

## Part II: Predictive data analysis

*Make predictions using structure found in Part I*

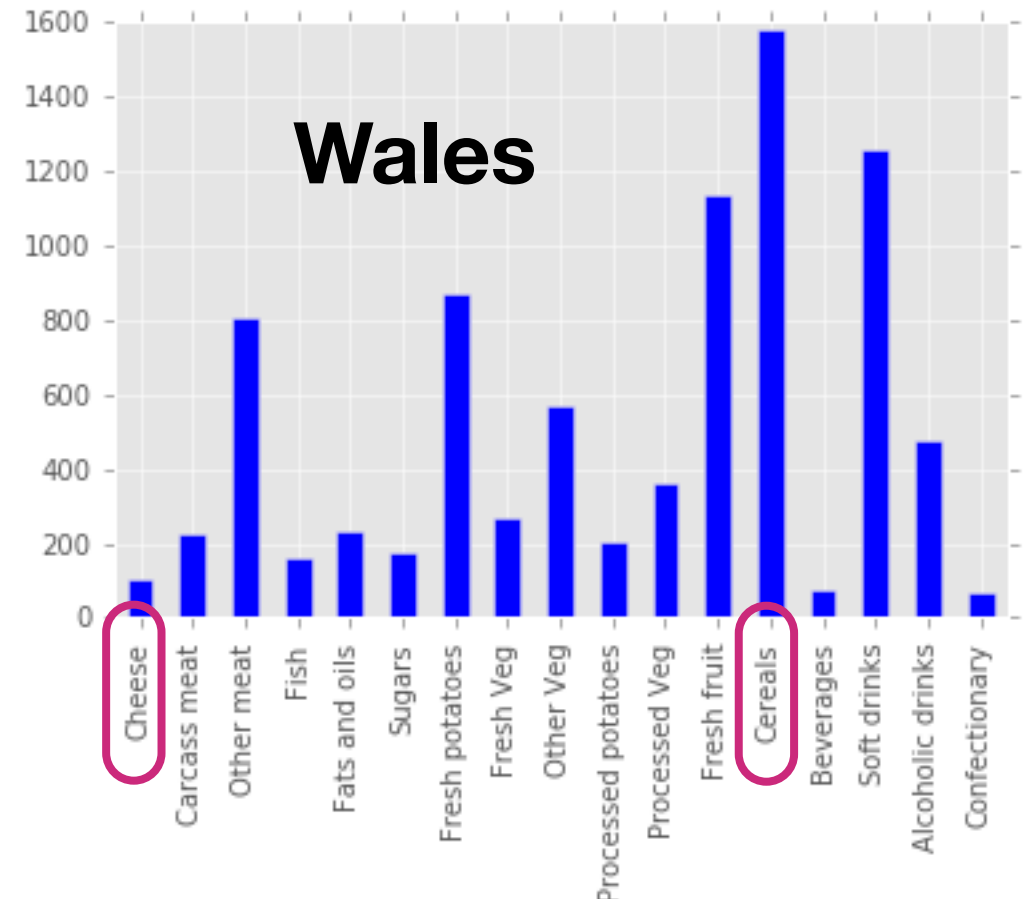
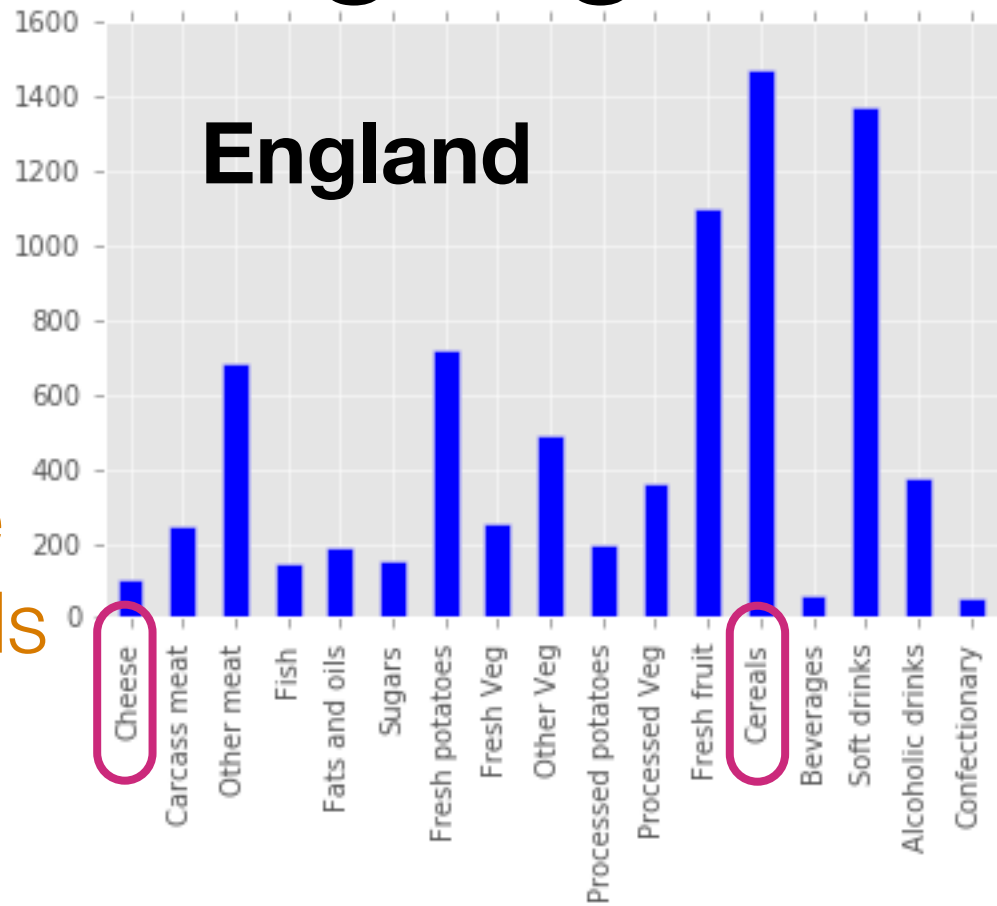
- Classical classification methods
- Neural nets and deep learning for analyzing images and text

# Visualizing High-Dimensional Vectors

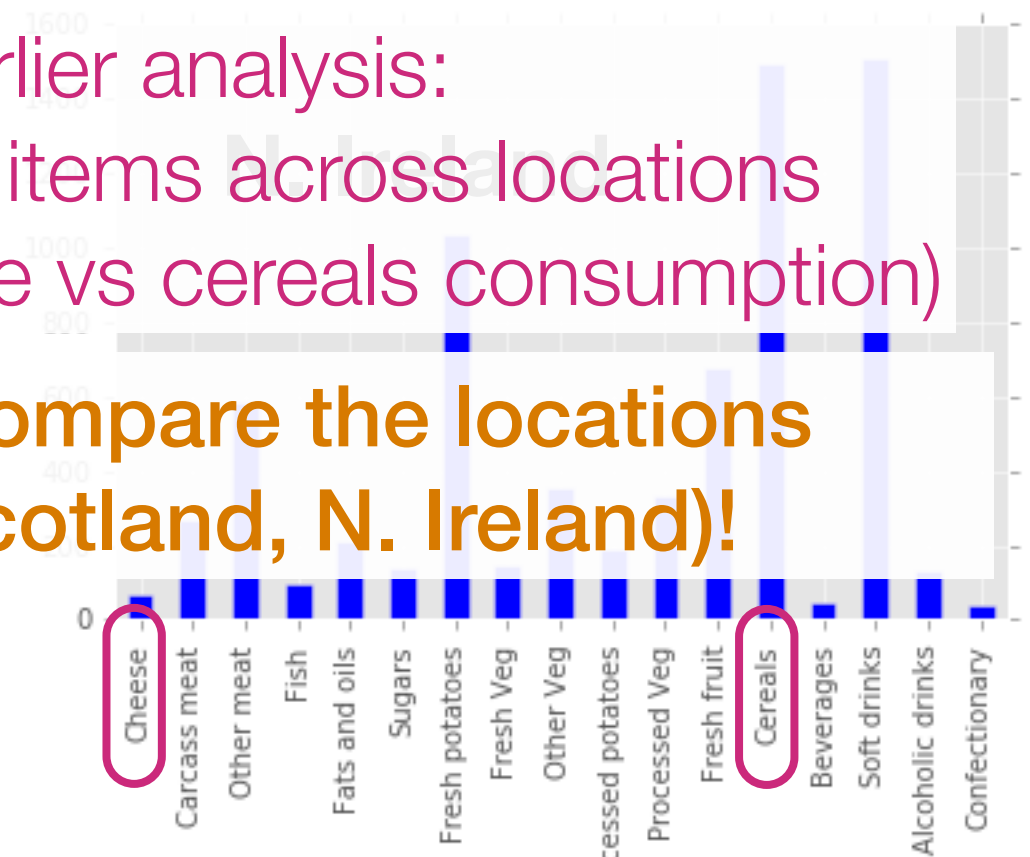
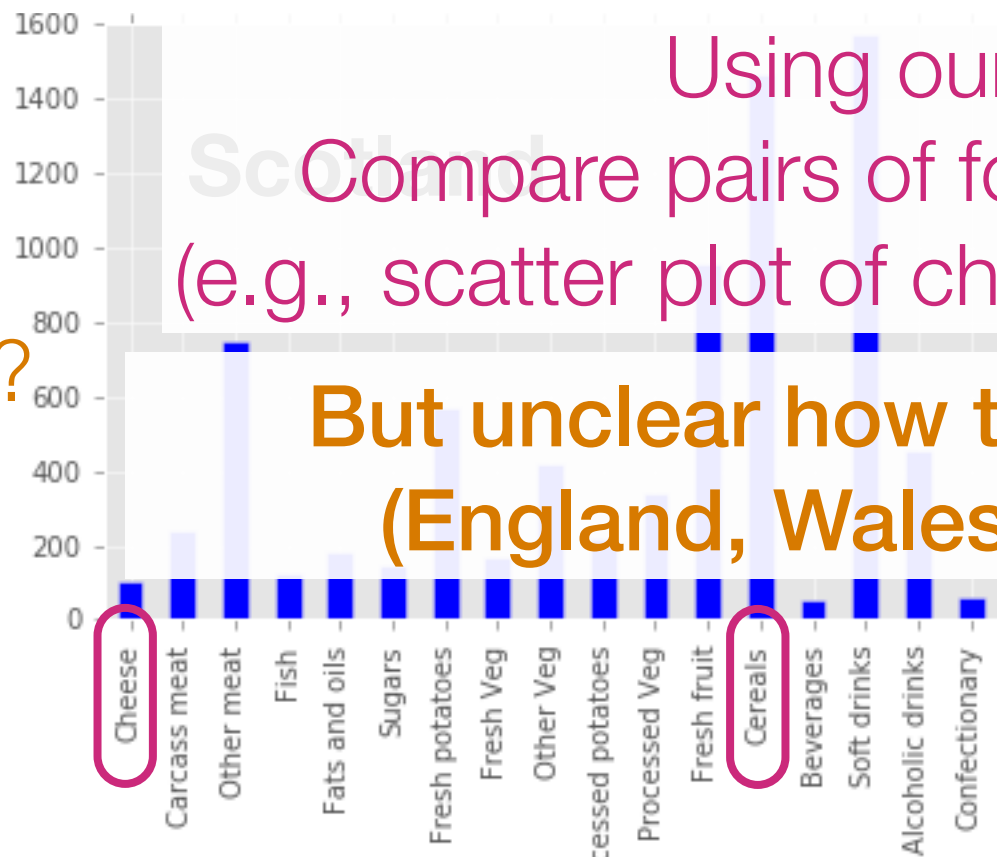
The next two examples are drawn from:  
<http://setosa.io/ev/principal-component-analysis/>

# Visualizing High-Dimensional Vectors

Imagine we had hundreds of these



How to visualize these for comparison?



Using our earlier analysis:  
Compare pairs of food items across locations  
(e.g., scatter plot of cheese vs cereals consumption)

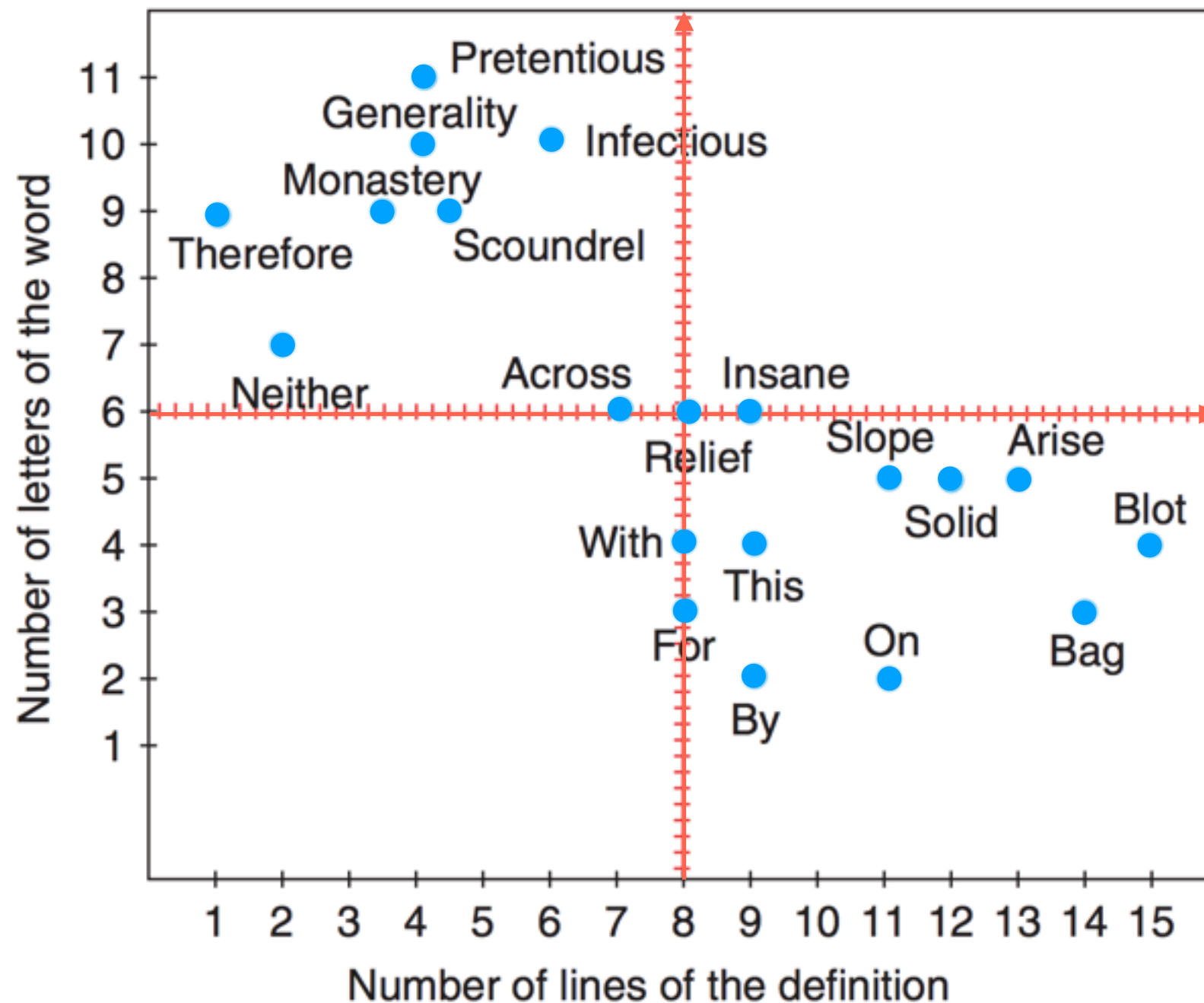
But unclear how to compare the locations  
(England, Wales, Scotland, N. Ireland)!

**The issue is that as humans  
we can only really visualize  
up to 3 dimensions easily**

Goal: Somehow reduce the dimensionality of the data  
preferably to 1, 2, or 3

# Principal Component Analysis (PCA)

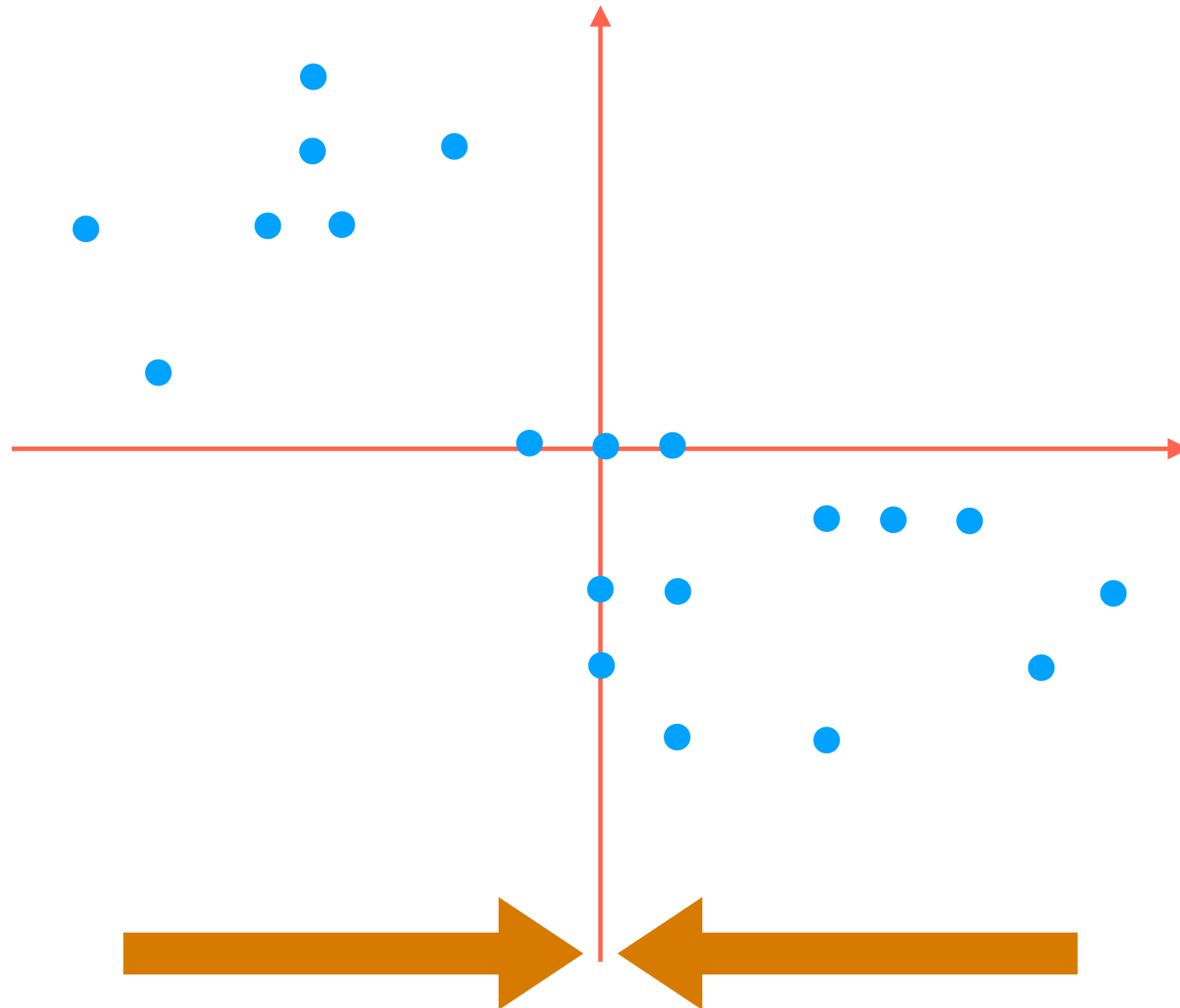
How to project 2D data down to 1D?





# Principal Component Analysis (PCA)

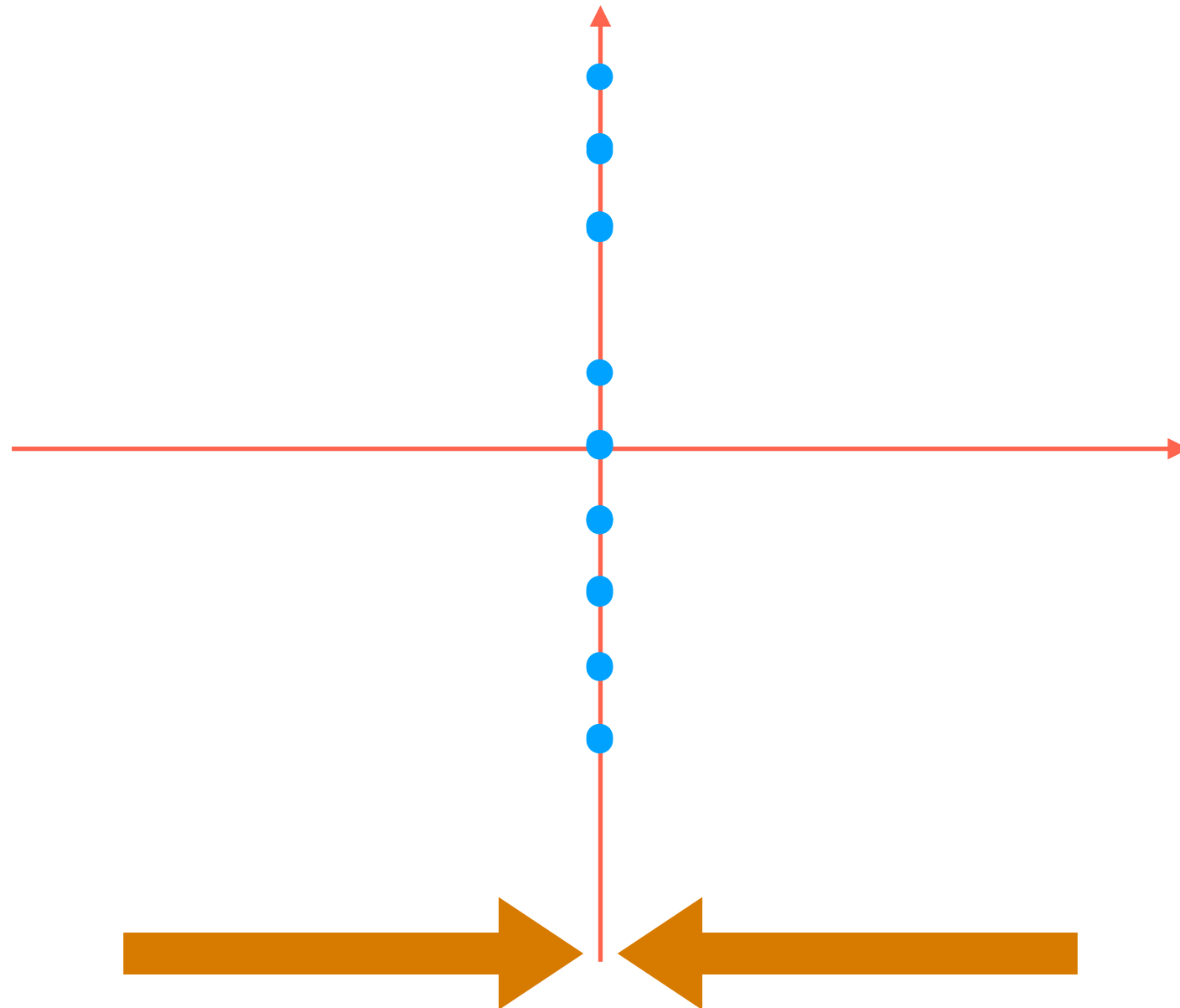
How to project 2D data down to 1D?



Simplest thing to try: flatten to one of the red axes

# Principal Component Analysis (PCA)

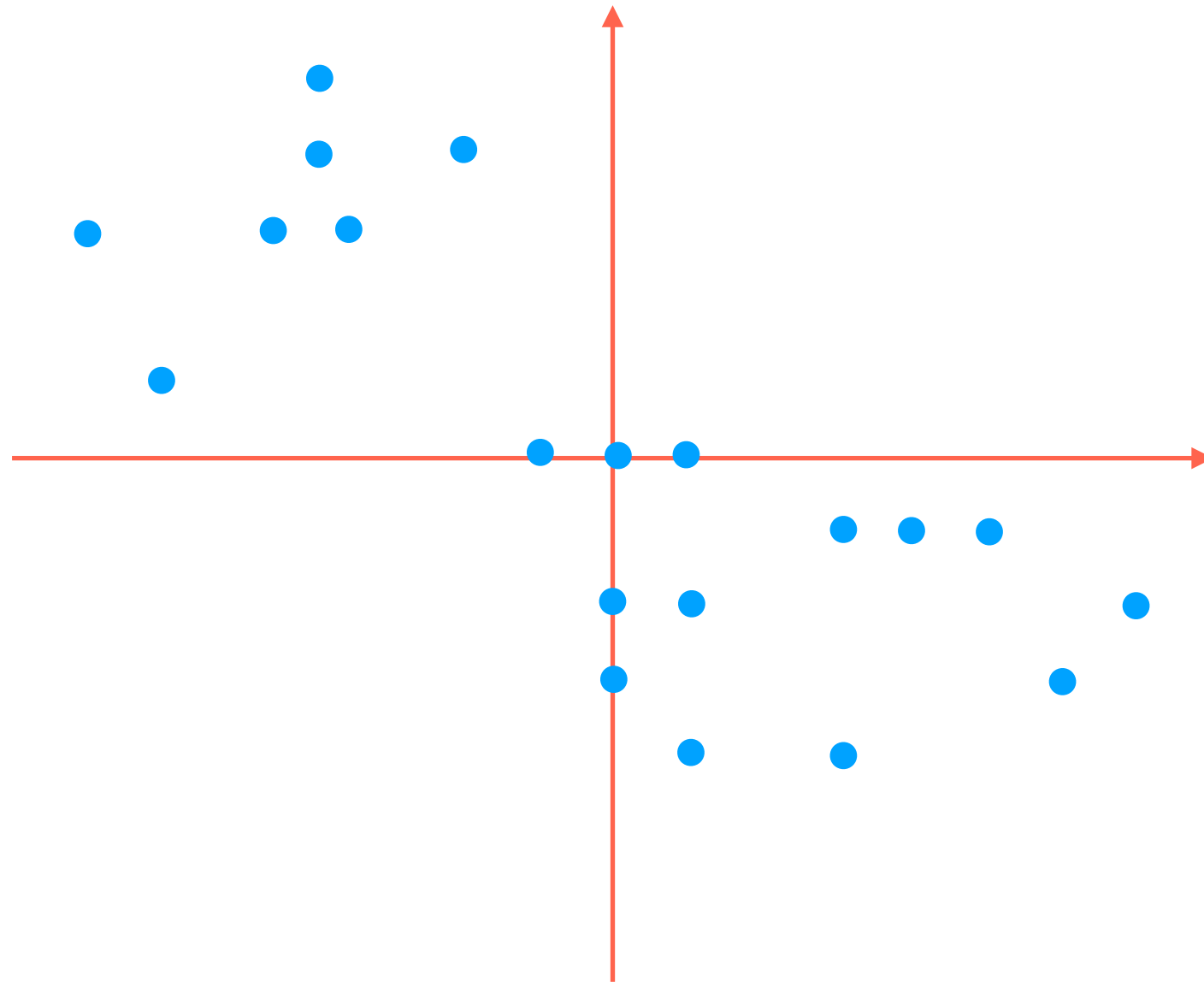
How to project 2D data down to 1D?



Simplest thing to try: flatten to one of the red axes  
(We could of course flatten to the other red axis)

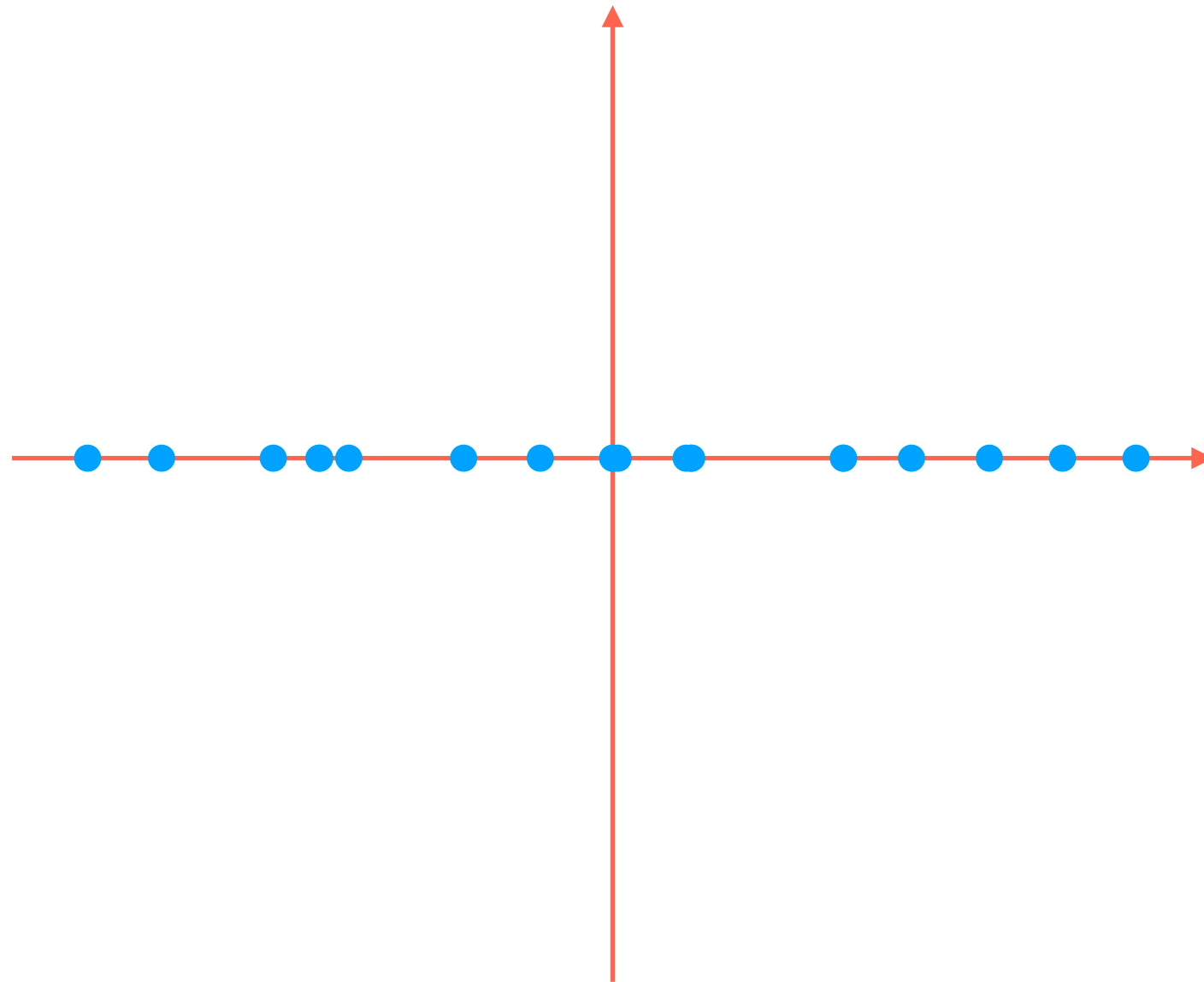
# Principal Component Analysis (PCA)

How to project 2D data down to 1D?



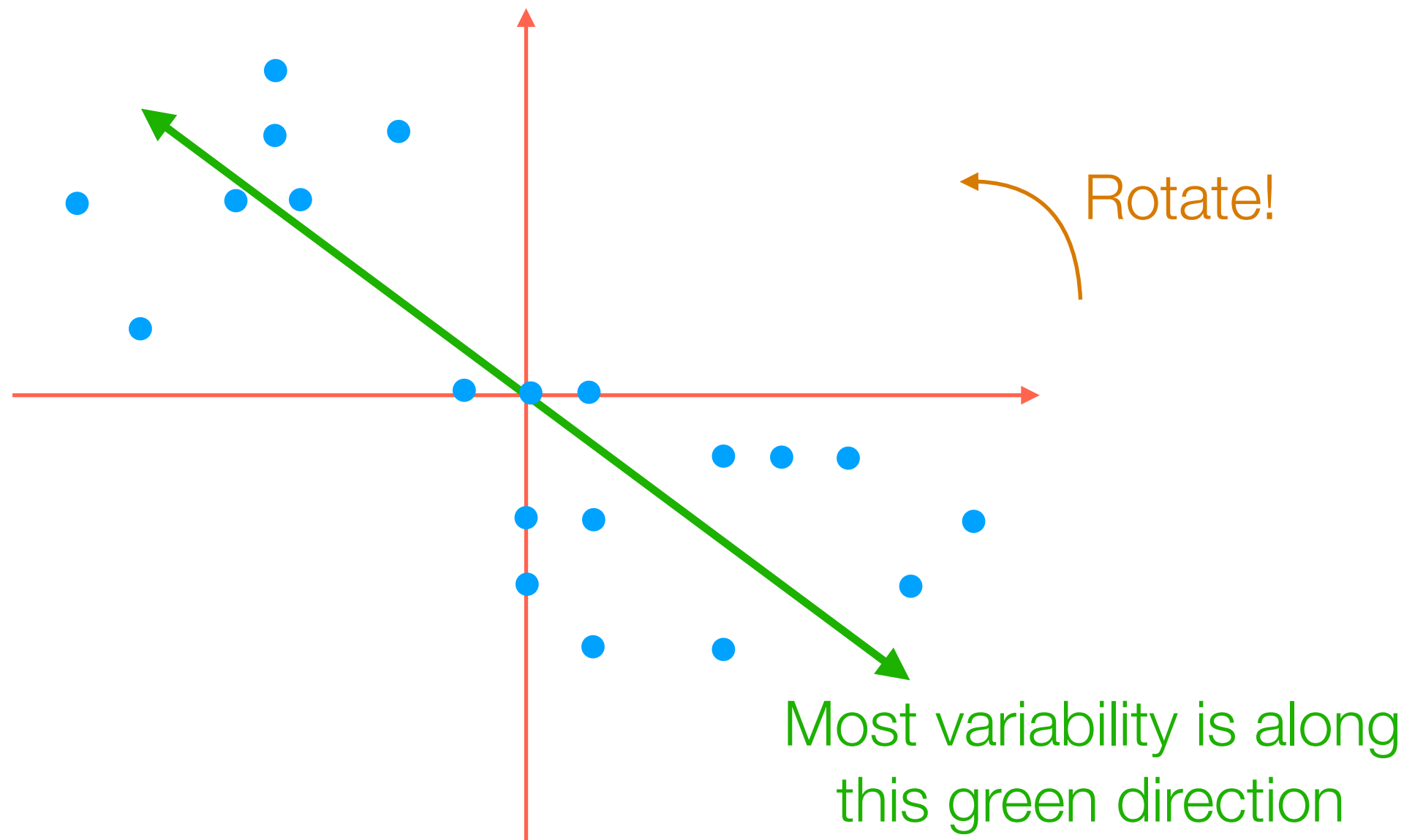
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# Principal Component Analysis (PCA)

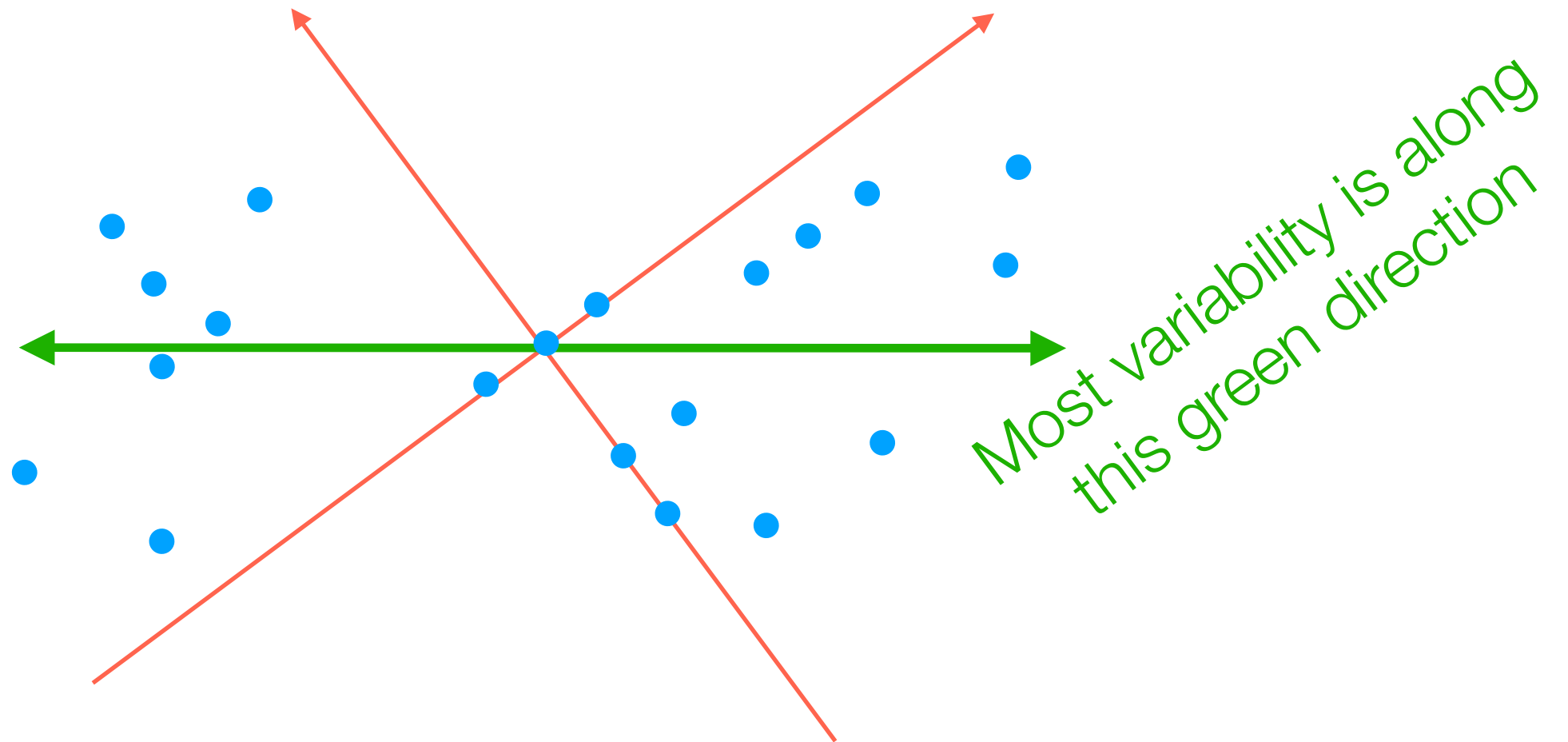
How to project 2D data down to 1D?



But notice that most of the variability in the data is *not* aligned with the red axes!

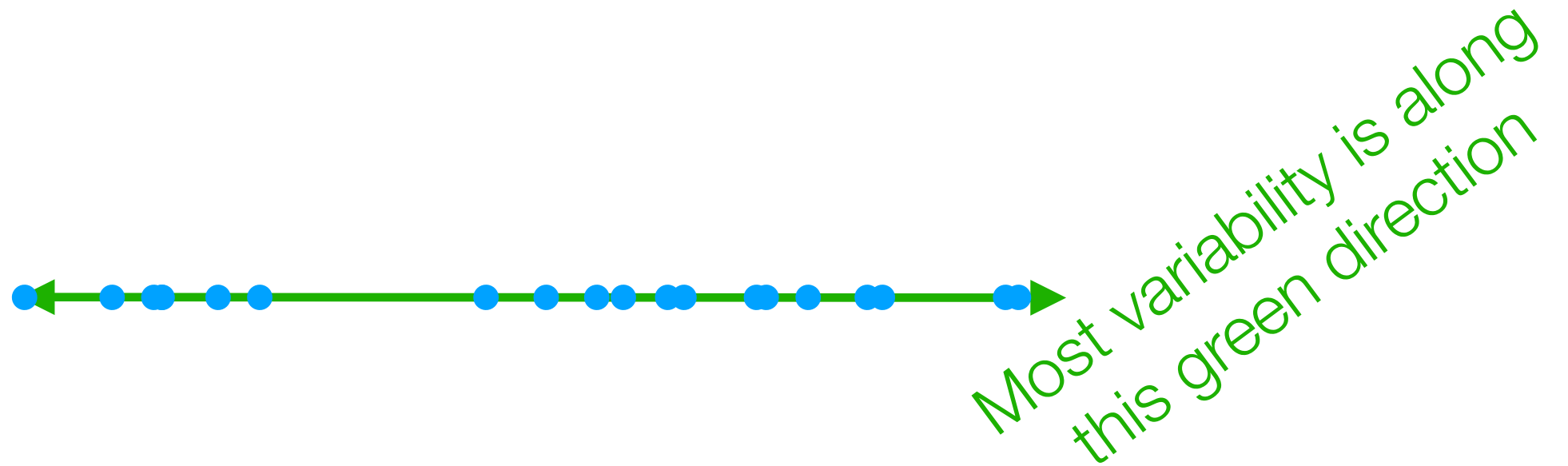
# Principal Component Analysis (PCA)

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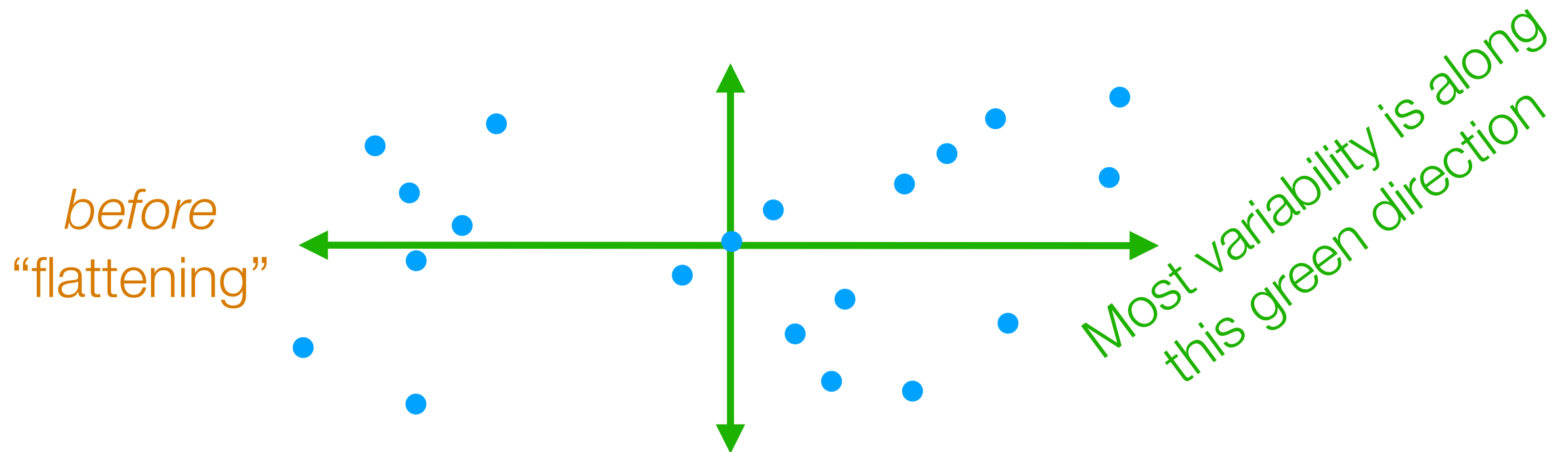


The idea of PCA actually works for 2D  $\rightarrow$  2D as well (and just involves rotating, and not “flattening” the data)

# Principal Component Analysis (PCA)

~~How to project 2D data down to 1D?~~

How to rotate 2D data so 1st axis has most variance



The idea of PCA actually works for  $2D \rightarrow 2D$  as well  
(and just involves rotating, and not "flattening" the data)

2nd green axis chosen to be  $90^\circ$  ("orthogonal") from first green axis



# Principal Component Analysis (PCA)

- Finds top  $k$  orthogonal directions that explain the most variance in the data
  - 1st component: explains most variance along 1 dimension
  - 2nd component: explains most of remaining variance along next dimension that is orthogonal to 1st dimension
  - ...
- “Flatten” data to the top  $k$  dimensions to get lower dimensional representation (if  $k <$  original dimension)

# Principal Component Analysis (PCA)

3D example from:

<http://setosa.io/ev/principal-component-analysis/>

# Principal Component Analysis (PCA)

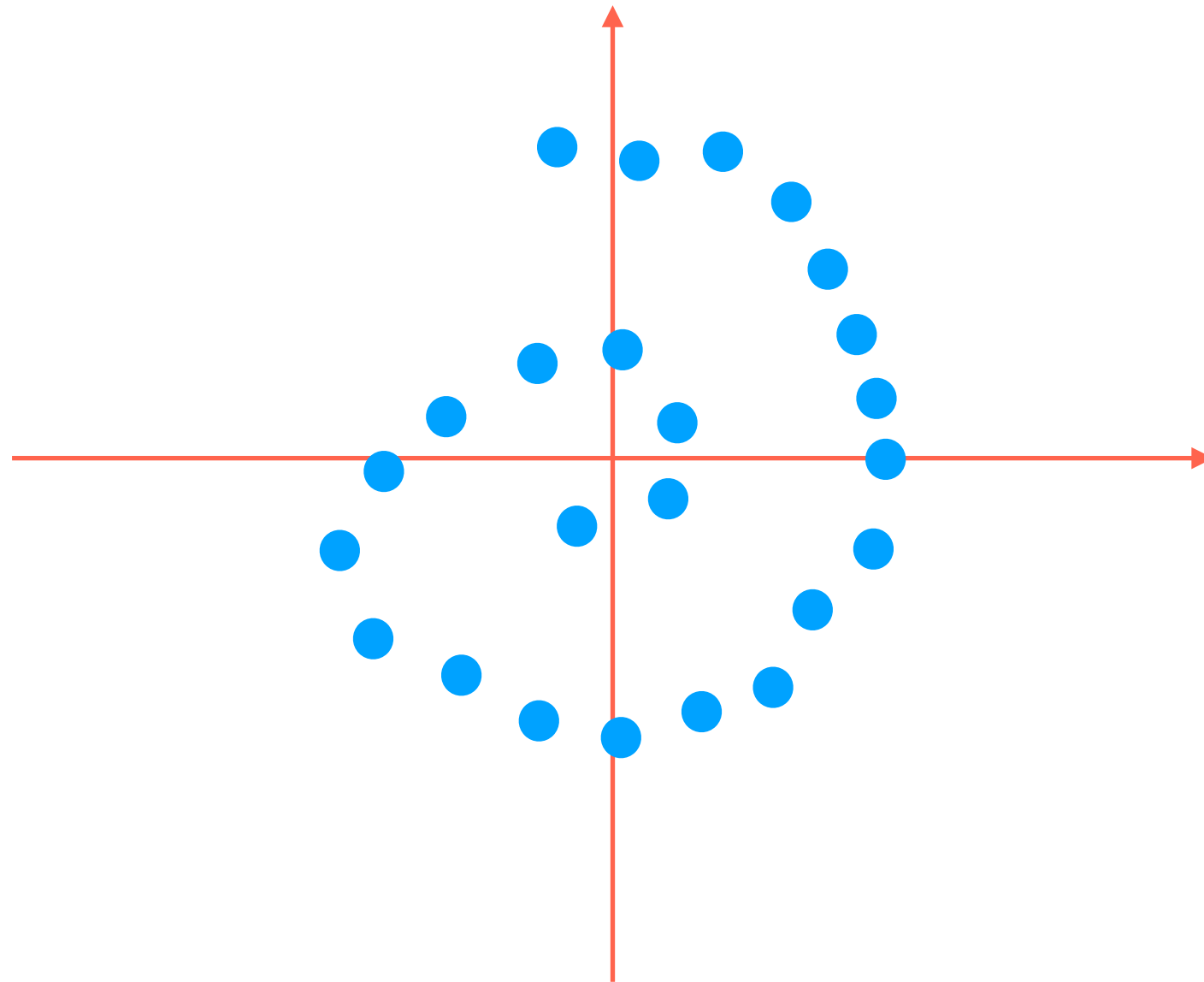
Demo

**PCA reorients data so axes explain variance in “decreasing order”**  
**→ can “flatten” (*project*) data onto a few axes that captures most variance**



Image source: [http://4.bp.blogspot.com/-USQEgoh1jCU/VfncdNOETcl/AAAAAAAAAGp8/Hea8UtE\\_1c0/s1600/Blog%2B1%2BIMG\\_1821.jpg](http://4.bp.blogspot.com/-USQEgoh1jCU/VfncdNOETcl/AAAAAAAAAGp8/Hea8UtE_1c0/s1600/Blog%2B1%2BIMG_1821.jpg)

# 2D Swiss Roll



PCA would just flatten this thing and  
*lose the information that the data actually  
lives on a 1D line that has been curved!*

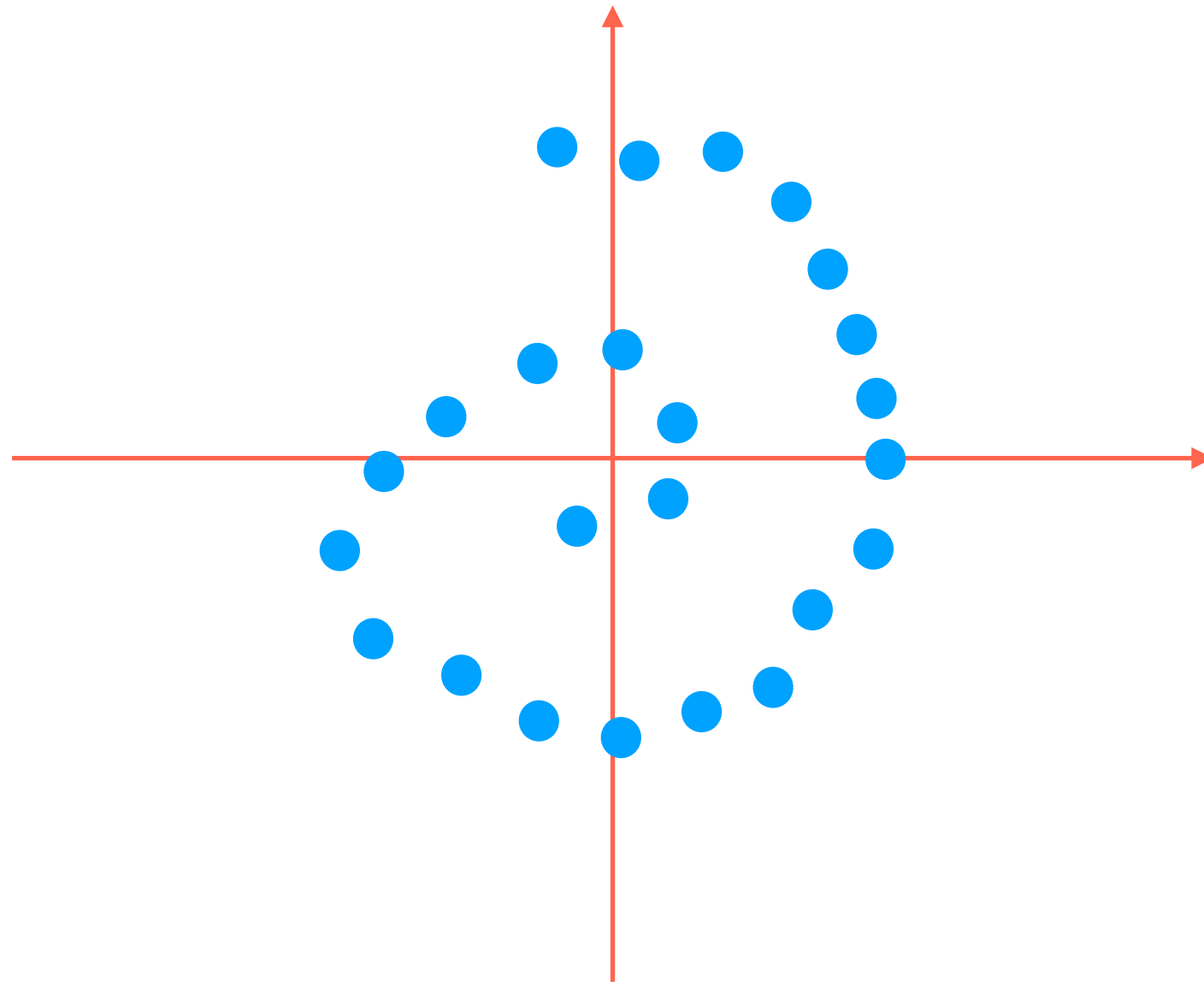




PCA would squash down this Swiss roll (like stepping on it from the top) mixing the red & white parts

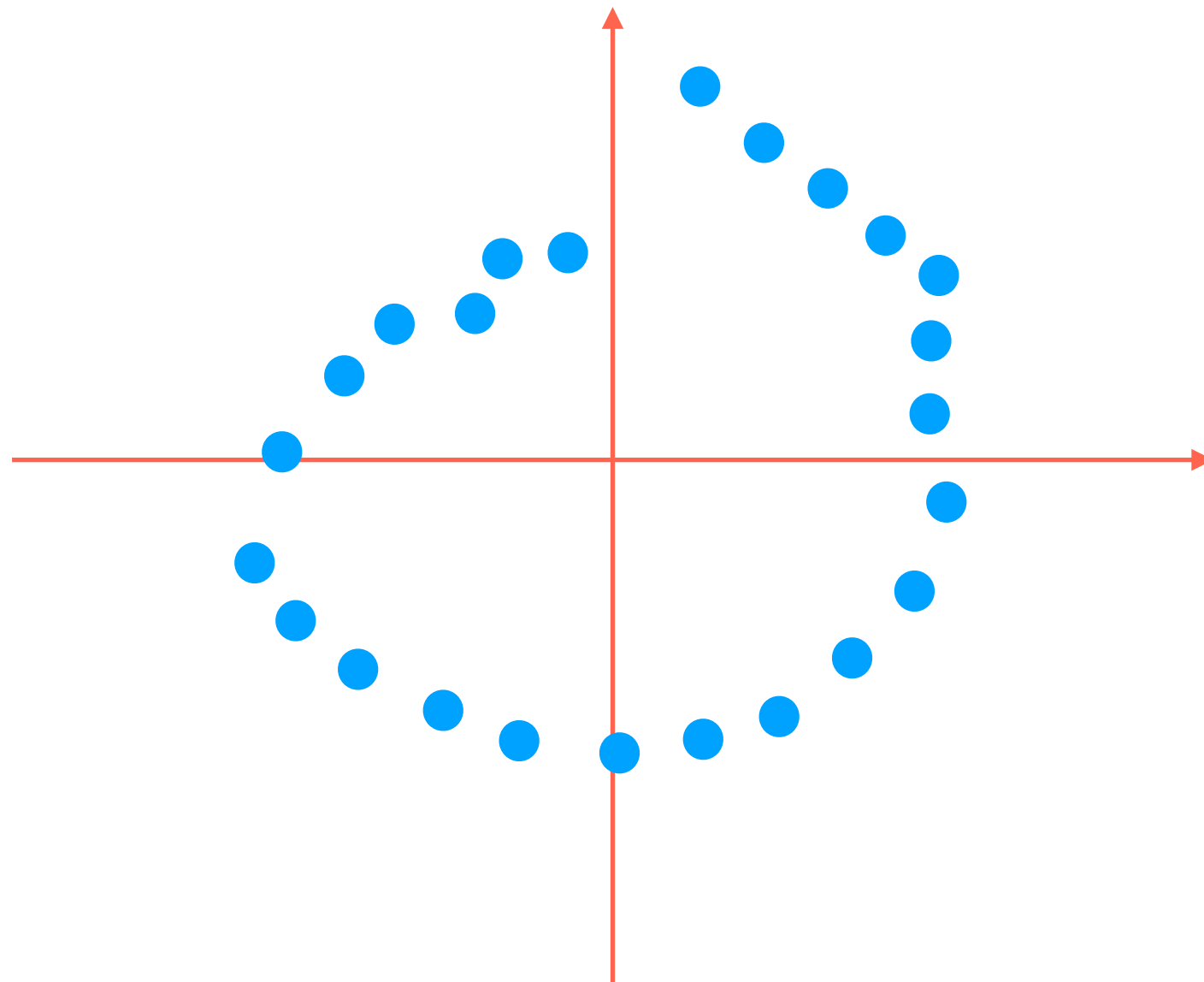
Image source: [http://4.bp.blogspot.com/-USQEgoh1jCU/VfncdNOETcl/AAAAAAAAAGp8/Hea8UtE\\_1c0/s1600/Blog%2B1%2BIMG\\_1821.jpg](http://4.bp.blogspot.com/-USQEgoh1jCU/VfncdNOETcl/AAAAAAAAAGp8/Hea8UtE_1c0/s1600/Blog%2B1%2BIMG_1821.jpg)

# 2D Swiss Roll

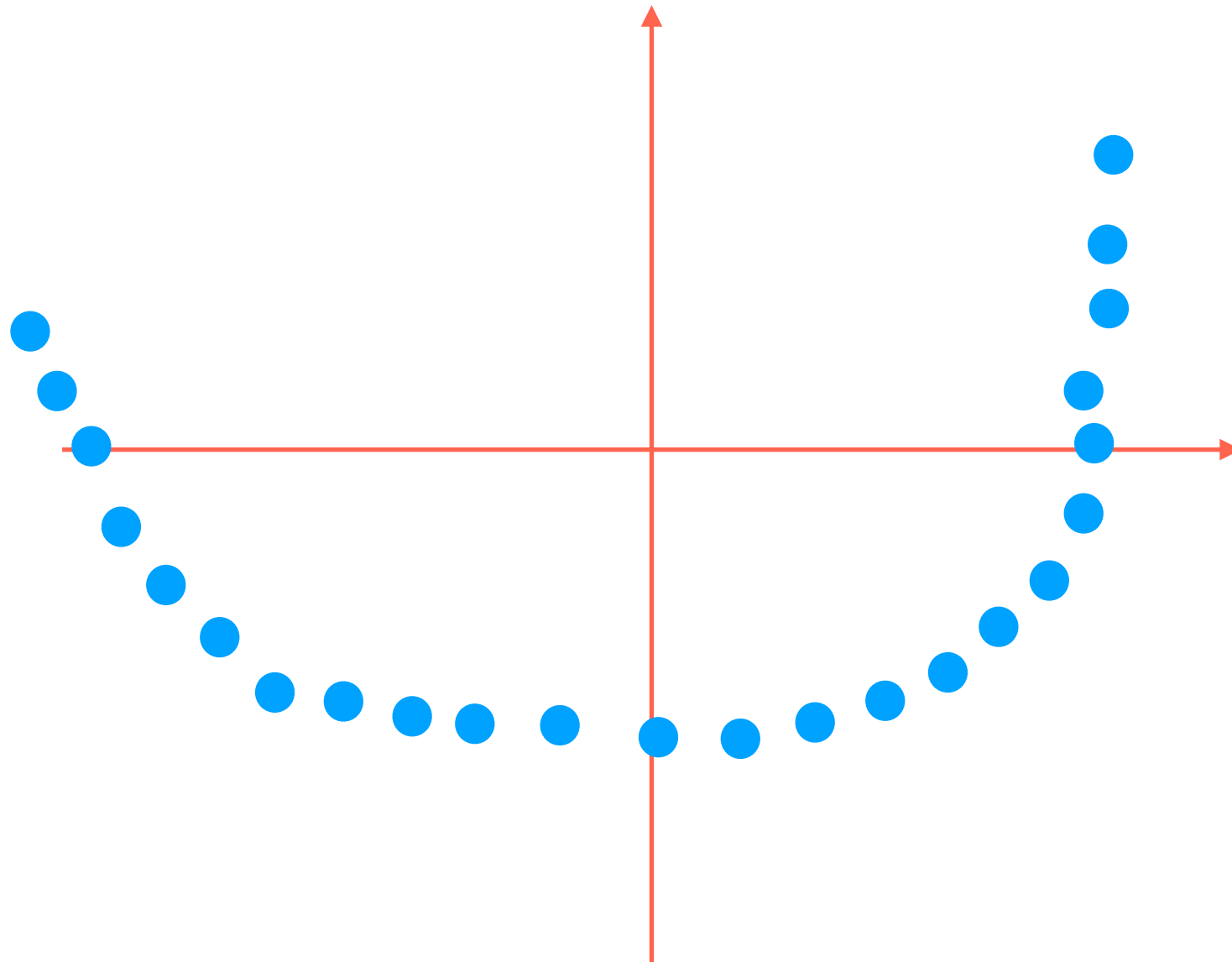




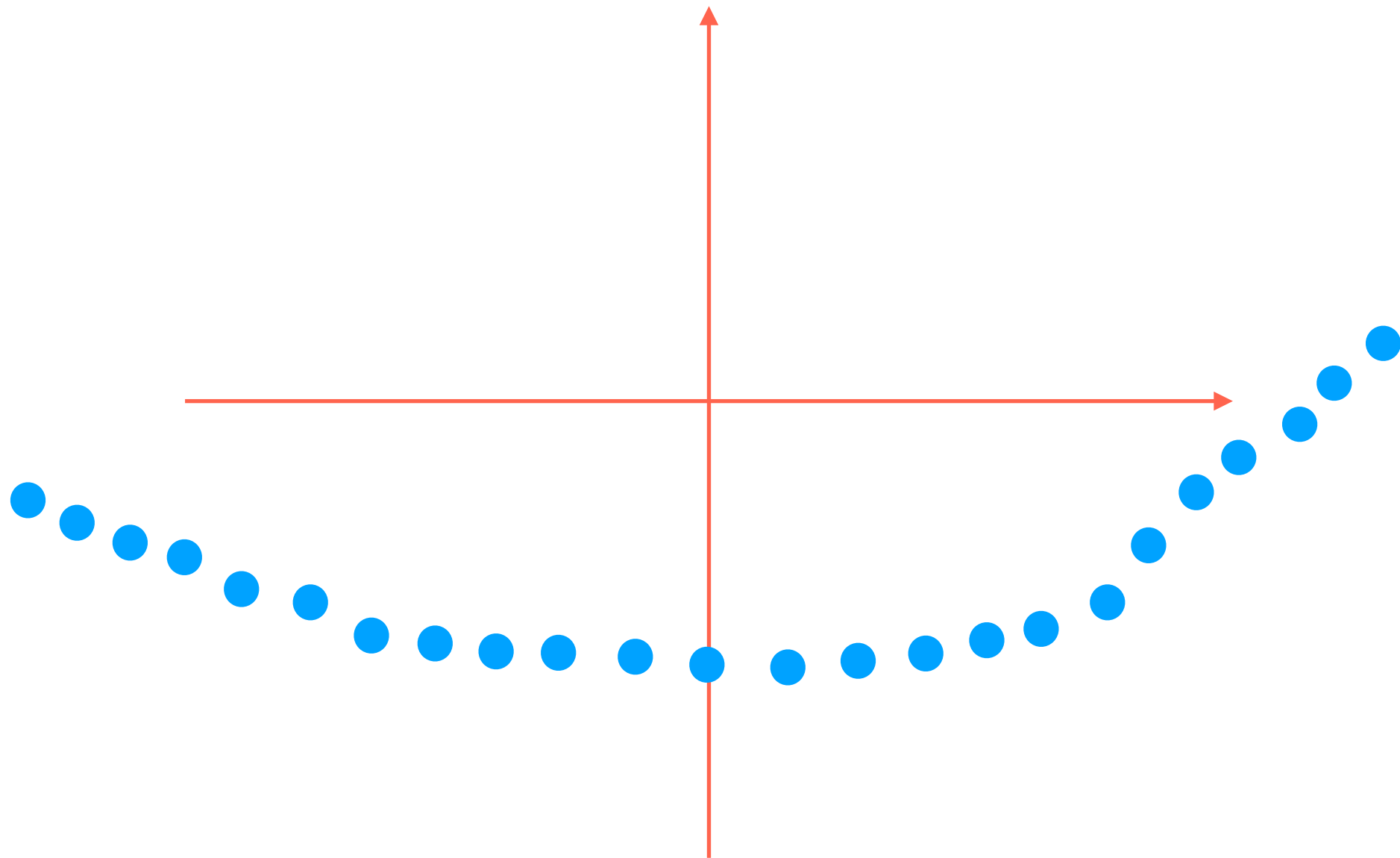
# 2D Swiss Roll



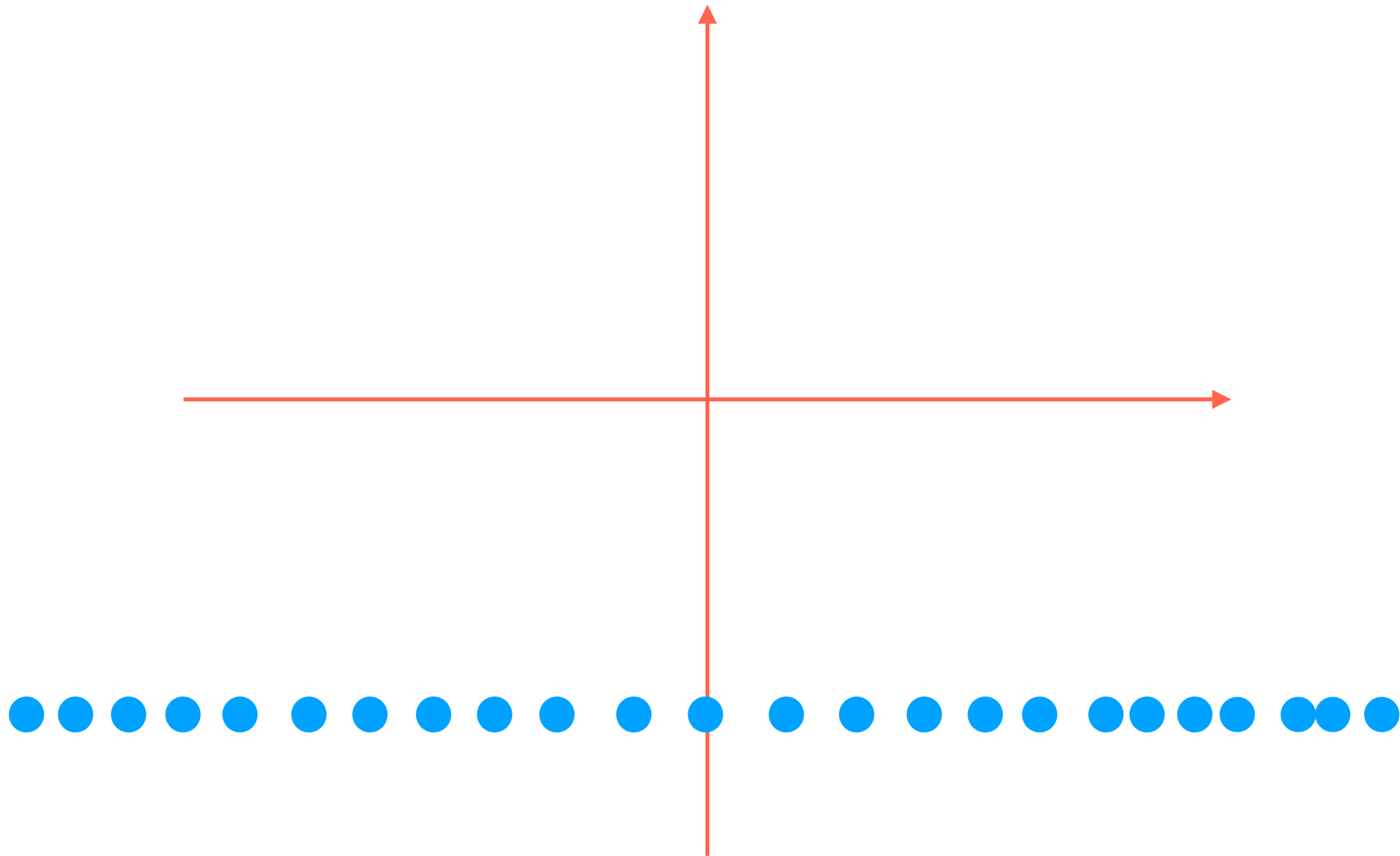
# 2D Swiss Roll



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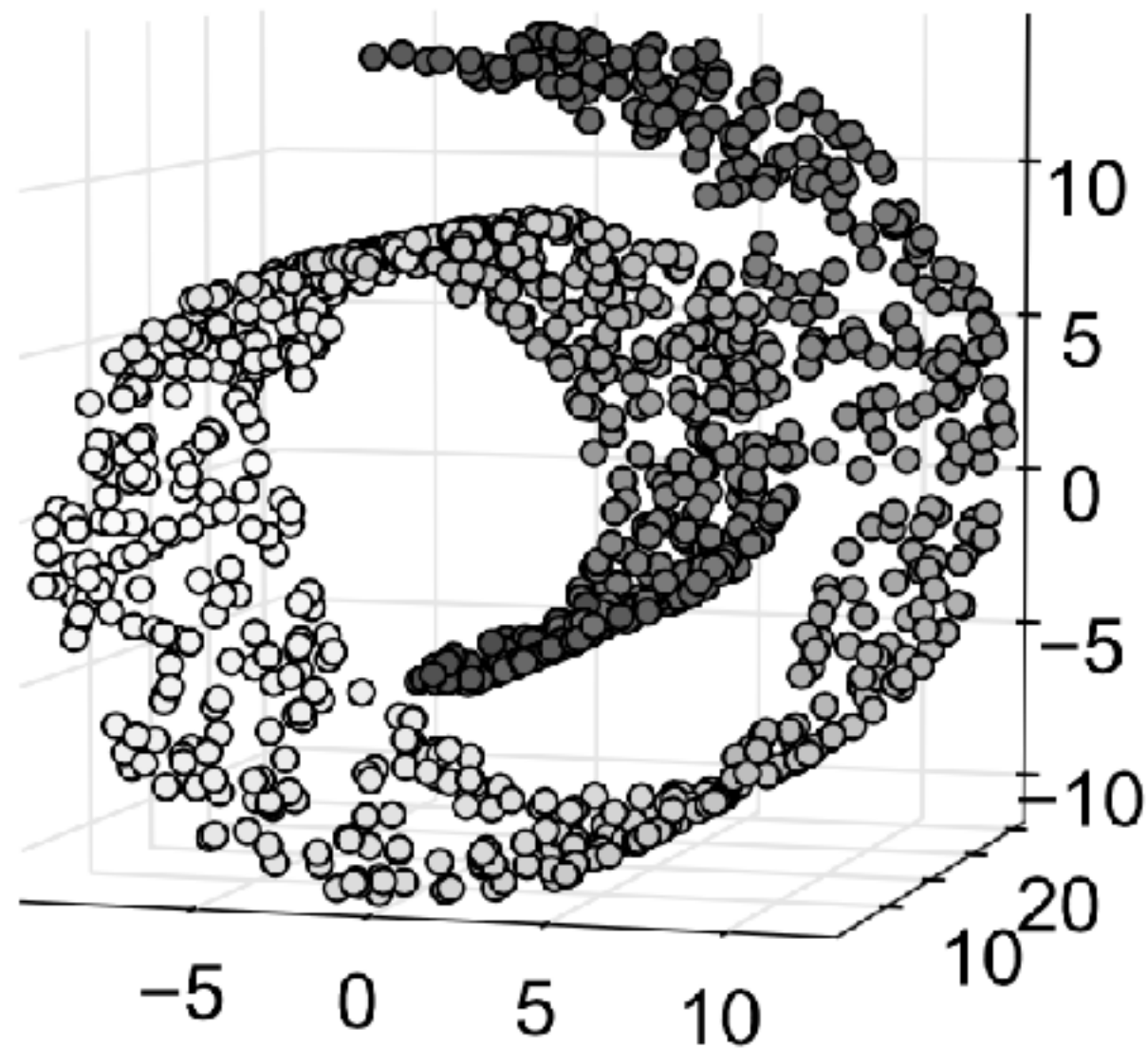


# 2D Swiss Roll



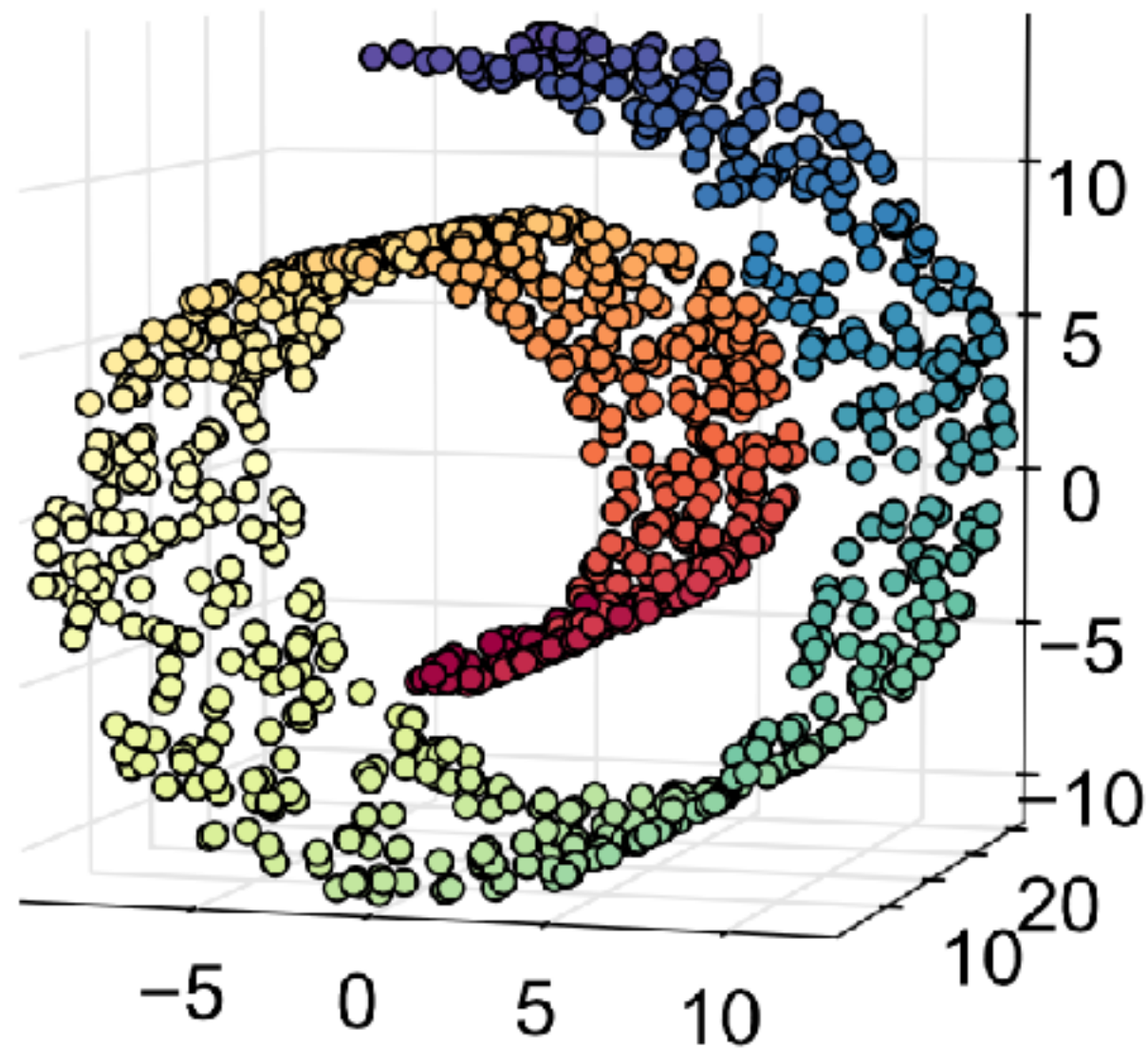
This is the desired result

# 3D Swiss Roll



Projecting down to any 2D plane puts points that are far apart close together!

# 3D Swiss Roll

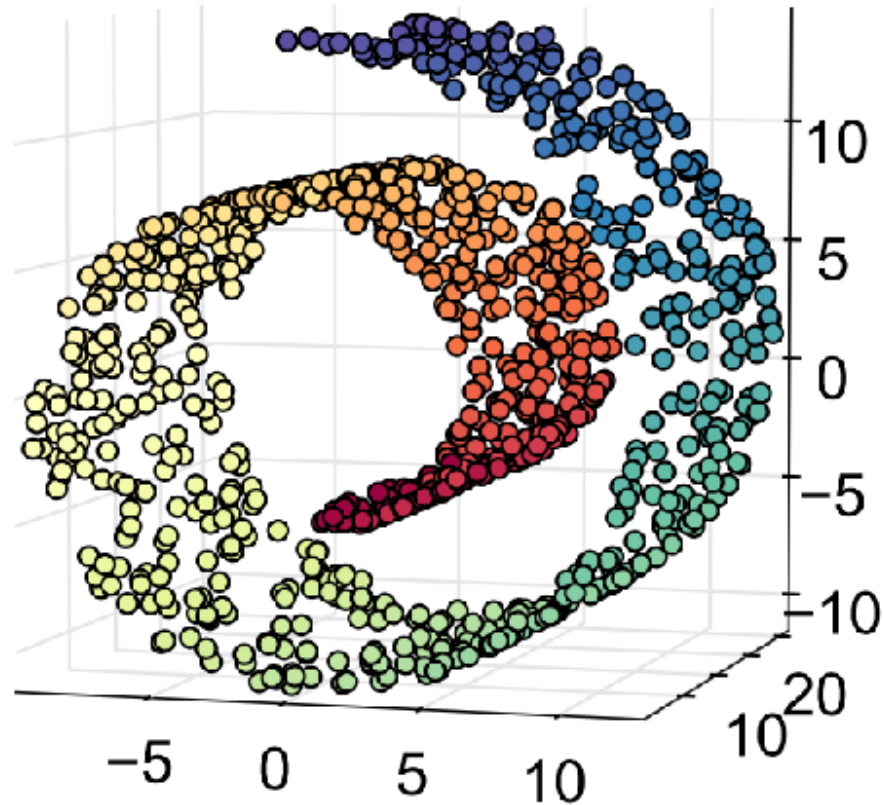


Projecting down to any 2D plane puts points that are far apart close together!

Goal: Low-dimensional representation where similar colored points are near each other (we don't actually get to see the colors)

# Manifold Learning

- Nonlinear dimensionality reduction (in contrast to PCA which is linear)
- Find low-dimensional “manifold” that the data live on



Basic idea of a manifold:

1. Zoom in on any point (say,  $x$ )
2. The points near  $x$  look like they're in a lower-dimensional Euclidean space (e.g., a 2D plane in Swiss roll)



# Do Data Actually Live on Manifolds?

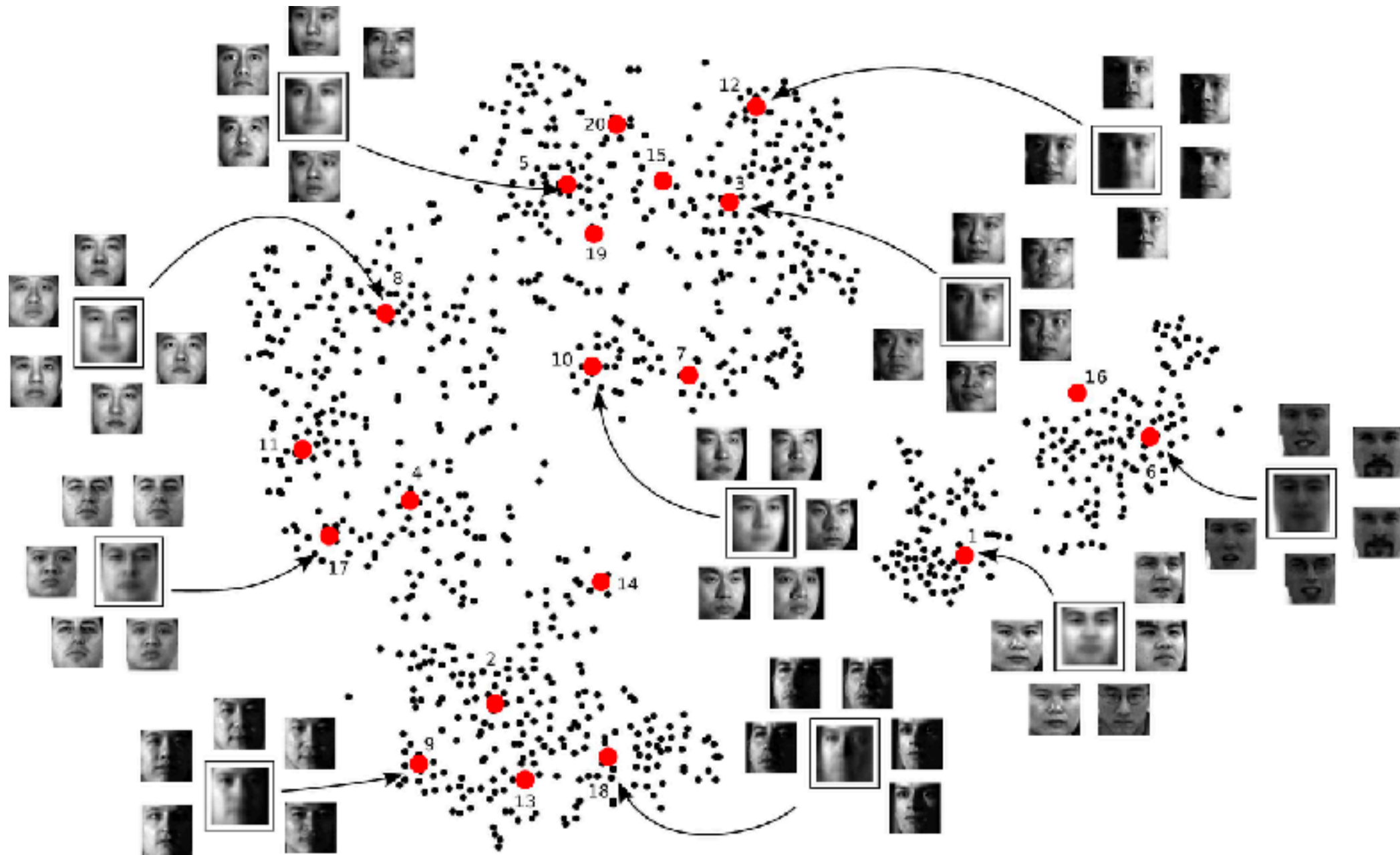


Image source: <http://www.columbia.edu/~jwp2128/Images/faces.jpeg>

# Do Data Actually Live on Manifolds?

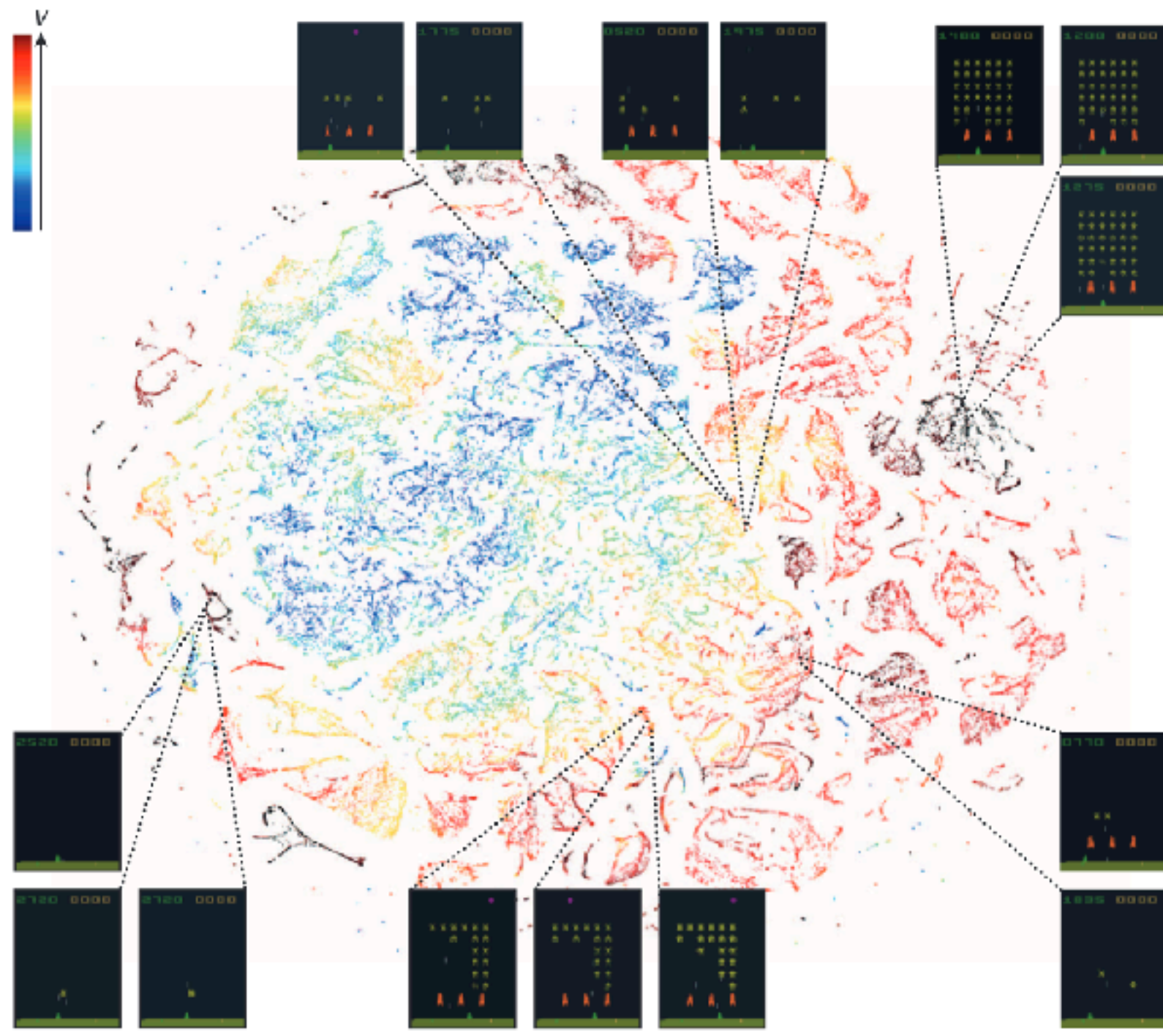


Phillip Isola, Joseph Lim, Edward H. Adelson. Discovering States and Transformations in Image Collections. CVPR 2015.





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Mnih, Volodymyr, et al. Human-level control through deep reinforcement learning. Nature 2015.